

$$f(n) = \sqrt{n+\epsilon} \sqrt{n-\epsilon} = \sqrt{n-\epsilon + \epsilon \sqrt{n-\epsilon} + \epsilon} = \sqrt{(\sqrt{n-\epsilon} + \epsilon)^2} = \sqrt{n-\epsilon} + \epsilon \quad (1)$$

$$g(n) = \sqrt{n-\epsilon} \sqrt{n-\epsilon} = \sqrt{n-\epsilon + \epsilon \sqrt{n-\epsilon} + \epsilon} = \sqrt{(\sqrt{n-\epsilon} - \epsilon)^2} = |\sqrt{n-\epsilon} - \epsilon| \xrightarrow{n \gg 1} \sqrt{n-\epsilon} - \epsilon$$

$$\rightarrow f(n) + g(n) = \sqrt{n-\epsilon} + \epsilon + \sqrt{n-\epsilon} - \epsilon = 2\sqrt{n-\epsilon} = a + b\sqrt{n+\epsilon}$$

$$\rightarrow a=0, b=2, c=-\epsilon \rightarrow \frac{a+b}{c} = \frac{0+2}{-\epsilon} = \boxed{-\frac{1}{\epsilon}}$$

$$f(n) = \frac{n+\epsilon}{n^2-\epsilon n+\epsilon^2}$$

(2)

$$g(n) = \{(1,1)(1,0)(\epsilon,0)(-\epsilon,0)\} \rightarrow D_g = \epsilon, 1, \epsilon, 0$$

$$f(\epsilon) = \frac{\epsilon}{-1} = -\epsilon \quad f(1) = \frac{\epsilon}{0} \propto \quad f(\epsilon) = \frac{0}{0} \propto \quad f(0) = \frac{\epsilon}{\epsilon}$$

$$\rightarrow \frac{g}{f} = \left\{ \left(-\frac{1}{\epsilon}, -\frac{1}{\epsilon} \right) \left(1, \frac{1}{\epsilon} \right) \left(\epsilon, \frac{1}{\epsilon} \right) \left(0, \frac{1}{\epsilon} \right) \right\} = \left\{ \left(\epsilon, -\frac{1}{\epsilon} \right) \left(0, \frac{1}{\epsilon} \right) \right\}$$

$$f(n) = \{(1,2)(\epsilon,-1)(\epsilon,2)(-1,2)\}$$

$$g(n) = \{(1,1)(\epsilon,2)(-2,2)(-1,0)\}$$

(3)

$$f(n) = \left\{ \left(\frac{1}{\epsilon}, 2 \right) \left(\frac{\epsilon}{\epsilon}, -1 \right) \left(\epsilon, 2 \right) \left(-\frac{1}{\epsilon}, 2 \right) \right\}$$

$$-) f(n^2) = \{(1,2)(-1,2)(\sqrt{\epsilon},-1)(-\sqrt{\epsilon},-1)(\epsilon,2)(-\epsilon,2)\}$$

$$ج) \epsilon g(n) + 1 = \{(1,2)(\epsilon,2)(-2,2)(-1,1)\}$$

$$د) \frac{\epsilon f}{g} = \left\{ (1,-\epsilon)(\epsilon,-1) \left(-\frac{1}{\epsilon}, \frac{\epsilon}{\epsilon} \right) \right\} = \{(1,-\epsilon)(\epsilon,-1)\}$$

$$f(n) = \{(1,1)(2,1)(3,1)(4,1)\}$$

$$g = \{(0,5)(1,5)(2,5)(3,5)(4,5)\}$$

$$h = f + g = \{(1,6)(2,6)(3,6)(4,6)\}$$

$$f(n) = \frac{\sqrt{r_n - n}}{n - r}$$

$$g(n) = [n] \times [-n]$$

$$\left(\frac{f}{g+1}\right)(n)$$

$$\begin{aligned} & \sqrt{r_n - n} \geq 0 \rightarrow \sqrt{r_n} \geq n \rightarrow r_n \geq n^2 \rightarrow n \leq r \\ & n \geq 0 \\ & n \neq r \rightarrow D_f = [-, r] - \{r\} \end{aligned}$$

$$g(n)+1: \dots \rightarrow D = \{0, 1, 2\}$$

$$f(n) = \sqrt{-n^2 + 4n - 4} = \sqrt{-(n^2 - 4n + 4)} = \sqrt{-(n-2)(n-2)} = \sqrt{-(n-2)^2} = |n-2| \rightarrow [1, 4]$$

$$g(n) = \{(1,2)(2,2)(3,2)(4,2)\}$$

$$h(n) = \frac{f(n)}{g(n)} = \left\{ \left(1, \frac{0}{2}\right) \left(2, \frac{\sqrt{0}}{2}\right) \left(3, \frac{0}{2}\right) \right\} = \left\{ \left(1, 0\right) \left(2, \frac{\sqrt{0}}{2}\right) \left(3, 0\right) \right\}$$

$$\begin{aligned} \Rightarrow y &= n^2 - 2n^2 + 2n \rightarrow y = n^2 - 2n^2 + 2n + 1 - 1 \rightarrow y = (n-1)^2 + 1 \\ \rightarrow y - 1 &= (n-1)^2 \rightarrow \sqrt{y-1} = n-1 \rightarrow \sqrt{y-1} + 1 = n \rightarrow R_f = \mathbb{R} \end{aligned}$$

$$\begin{aligned} \Rightarrow y &= \frac{1}{n^2 - 2n} \rightarrow y n^2 - 2y n - 1 = 0 \rightarrow n = \frac{2y \pm \sqrt{4y^2 + 4y}}{2y} \rightarrow 4y^2 + 4y \geq 0 \\ &\rightarrow 4y(y+1) \geq 0 \rightarrow \frac{1}{+ \frac{1}{4} +} \rightarrow R_f = (-\infty, -1] \cup [0, +\infty) \end{aligned}$$

$$\rightarrow 1) y = x^r - tx^{r+1} + 1 \rightarrow \min \left| \begin{array}{l} \frac{f}{x} = r \\ t - (r+1)x = 0 \end{array} \right. \rightarrow R_f = [r, +\infty)$$

①

$$\rightarrow 2) y = -x^r + tx^{r+1} + r \rightarrow \max \left| \begin{array}{l} \frac{f}{x} = r \\ -r + (r+1)x = 1 \end{array} \right. \rightarrow R_f = (-\infty, 1r]$$

$$3) y = \sqrt{x^r - tx^{r+1}} \rightarrow \min \left| \begin{array}{l} \frac{f}{x} = r \\ t - (r+1)x = -1 \end{array} \right. \rightarrow R_f = \sqrt{[-1, +\infty)} = [0, +\infty)$$

$$4) y = \sqrt{-x^r + tx^{r+1}} \rightarrow \max \left| \begin{array}{l} \frac{f}{x} = r \\ -r + (r+1)x = 1 \end{array} \right. \rightarrow R_f = \sqrt{(-\infty, 1]} = [0, 1r]$$

$$\rightarrow 5) y = x^r - 2x^{r+1} + x^{r+1} \rightarrow R_f = \mathbb{R}$$

②

$$\rightarrow 6) y = x^r - 2x^{r+1} + x^{r+1} \rightarrow R_f = \mathbb{R}$$

$$7) y = \sqrt{x^r - 2x^{r+1} + x^{r+1}} \rightarrow R_f = \sqrt{\mathbb{R}} = [0, +\infty)$$

$$8) y = (x^r - 2x^{r+1} + x^{r+1})^r \rightarrow R_f = \mathbb{R}^+ = [0, +\infty)$$

$$\rightarrow 9) y = \frac{x^{r+1}}{x-r} \rightarrow R_f = \mathbb{R} - \{r\}$$

③

$$\rightarrow 10) y = \frac{x^{r+1}}{x+1} \rightarrow R_f = \mathbb{R} - \{-1\}$$