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$$f(x) = \sqrt{x+4\sqrt{x-4}} \Rightarrow (\sqrt{x-4})^2 + x-4 \rightarrow x = (\sqrt{x-4})^2 + 4 \rightarrow x+4\sqrt{x-4} = (\sqrt{x-4})^2 + 4 + 4\sqrt{x-4} \Rightarrow (\sqrt{x-4} + 2)^2$$

$$g(x) = \sqrt{x-4\sqrt{x-4}} \Rightarrow x-4\sqrt{x-4} = (\sqrt{x-4} - 2)^2$$

$$\rightarrow f(x) = \sqrt{(\sqrt{x-4} + 2)^2} \xrightarrow{x \geq 4} \sqrt{x-4} + 2 = f(x) \Rightarrow f(x) + g(x) = 2\sqrt{x-4} = a + b\sqrt{x} + c$$

$$\Rightarrow f(x) + g(x) = 2\sqrt{x-4} = a + b\sqrt{x} + c$$

با جابجایی

$$a=0, b=2, c=-4$$

$$\frac{a+b}{c} = \frac{1}{1/2}$$

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$$f(x) = \frac{x+2}{x^2-4x+3} \rightarrow D_f = \mathbb{R} \setminus \{1, 3\}$$

$$g(x) = \{(1,1), (1,0), (2,1), (0,2)\} \rightarrow D_g = \{1, 1, 2, 0\}$$

$$\frac{g}{f} \xrightarrow{x=1} \frac{1}{\frac{1}{-1}} = -1$$

$$\frac{g}{f} \xrightarrow{x=0} \frac{2}{\frac{3}{-3}} = -2$$

$$\Rightarrow \frac{g}{f} = \{(1, -1), (0, -2)\}$$

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$$f(x) = \{(1,2), (2,1), (4,2), (-1,4)\} \rightarrow D_f = \{1, 2, 4, -1\}$$

$$g(x) = \{(1,2), (1,-1), (2,2), (-1,0)\} \rightarrow D_g = \{1, 1, 2, -1\}$$

$$\text{inf } f(x) = \begin{cases} x=1 \rightarrow x=1/4 \\ x=2 \rightarrow x=2/4 \\ x=4 \rightarrow x=4/4 \\ x=-1 \rightarrow x=-1/4 \end{cases} \Rightarrow \{(1/4, 2), (2/4, 1), (4/4, 2), (-1/4, 4)\}$$

$$f(x) = \begin{cases} x^2=1 \rightarrow x=\pm 1 \\ x^2=2 \rightarrow x=\pm\sqrt{2} \\ x^2=4 \rightarrow x=\pm 2 \\ x^2=-1 \rightarrow \text{no real solution} \end{cases} \Rightarrow \{(1,2), (-1,2), (1,-1), (-1,-1), (2,2), (-2,2), (2,-1), (-2,-1)\}$$

$$2) \frac{g}{f}(x) + 1 \Rightarrow \begin{cases} x=-2 \rightarrow 2+9+1=12 \\ x=1 \rightarrow 2+1=3 \\ x=2 \rightarrow 4+1=5 \\ x=-1 \rightarrow 0+1=1 \end{cases} \Rightarrow \{(-2,12), (1,3), (2,5), (-1,1)\}$$

$$\frac{g}{f} = \begin{cases} x=1 \rightarrow \frac{2}{-1} = -2 \\ x=2 \rightarrow \frac{2}{-1} = -2 \\ x=-1 \rightarrow \frac{2}{-1} = -2 \end{cases} \Rightarrow \{(1,-2), (2,-2), (-1,-2)\}$$

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$$f(x) = \{(1,1), (2,1), (4,9), (-1,0)\} \rightarrow D_f = \{1, 2, 4, -1\}$$

$$g(x) = \{(1,1), (2,4), (-1,4), (-1,2), (2,4)\} \rightarrow D_g = \{1, 2, -1, -1, 2\} \Rightarrow \{1, 2, -1\}$$

$$h = 2f + g$$

$$\Rightarrow \begin{cases} x=2 \rightarrow 2+4=6 \\ x=3 \rightarrow 2+9=11 \\ x=4 \rightarrow 0+4=4 \end{cases} \Rightarrow \{(2,6), (3,11), (4,4)\}$$

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$$f(x) = \sqrt{4x-x} \rightarrow D_f = [0, 4] \setminus \{1\}$$

$$g(x) = [x] + [-x] \rightarrow D_g = \mathbb{Z} \Rightarrow \{0, 1, 2\}$$

$$\begin{cases} x \geq 0 \rightarrow x \geq 0 \\ \sqrt{4x-x} \geq 0 \rightarrow \sqrt{4x-x} \geq 0 \\ x \geq 0 \rightarrow x \geq 0 \\ x \geq 0 \rightarrow x \geq 0 \end{cases}$$

$$\left(\frac{f}{g+1}\right)(x) = \frac{f(x)}{g(x)+1} \Rightarrow \begin{cases} x \geq 0 \rightarrow \frac{2}{-1} = -2 \\ x=1 \rightarrow \frac{1}{-1} = -1 \\ x=2 \rightarrow \frac{1}{-1} = -1 \end{cases} \Rightarrow \{(0,0), (1,-1), (2,-1)\}$$

$$f(u) = \sqrt{4u - u^2 - 4}$$

$$4u - u^2 - 4 \geq 0$$

$$u^2 - 4u + 4 \leq 0$$

$$(u-1)(u-4) \leq 0$$

$$g(u) = \{(1, 2), (2, 2), (4, 2), (4, 1)\}$$

$$L D g = \{(1, 2), (2, 2)\} \xrightarrow{\wedge} \{(1, 2), 2\}$$

$$h(u) = \frac{f(u)}{g(u)} = \begin{cases} u=1 \rightarrow \frac{0}{2} = 0 \\ u=2 \rightarrow \frac{\sqrt{4}}{2} = 1 \\ u=4 \rightarrow \frac{0}{2} = 0 \end{cases} \Rightarrow \{(1, 0), (2, 1), (4, 0)\}$$

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$$\text{الف) } y = x^3 - 3x^2 + 3x \Rightarrow y = (x-1)^3 + 1 \rightarrow R_f = \mathbb{R}$$

$$\text{ب) } y = \frac{1}{x^2 - 2x} \Rightarrow x^2 y - 2xy = 1 \rightarrow y(x^2 - 2x) = 1 \rightarrow yx^2 - 2xy - 1 = 0 \xrightarrow{\Delta \geq 0} \begin{aligned} &4y^2 - 4y \geq 0 \\ &4y(y-1) \geq 0 \\ &\frac{y-1}{y-0} \geq 0 \end{aligned}$$

$$R_f = (-\infty, -1] \cup (0, \infty)$$

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$$\text{الف) } y = x^2 - 4x + 1$$

$$\frac{-b}{2a} = \frac{4}{2} = 2 \rightarrow x = 2 \rightarrow y = -3 \rightarrow R_f = [-3, \infty)$$

$$\text{ب) } y = \sqrt{x^2 - 4x - 3}$$

$$\frac{-b}{2a} = \frac{2}{1} = 2 \rightarrow x = 2 \rightarrow y = -5 \rightarrow R_f = [-5, \infty)$$

$$\text{ب) } y = -x^2 + 4x + 3$$

$$\frac{-b}{2a} = \frac{-4}{-2} = 2 \rightarrow x = 2 \rightarrow y = 7 \rightarrow R_f = (-\infty, 7]$$

$$\text{د) } y = \sqrt{4x - x^2}$$

$$\frac{-b}{2a} = \frac{-4}{-2} = 2 \rightarrow x = 2 \rightarrow y = 2 \rightarrow R_f = [0, 2]$$

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$$\text{الف) } y = x^3 - 4x^2 + 3x + 1 \rightarrow R_f = \mathbb{R}$$

$$\text{ب) } y = x^3 - 4x^2 + 3x - 2 \rightarrow R_f = \mathbb{R}$$

$$\text{ج) } y = \sqrt{x^3 - 4x^2 + 3x + 1} \rightarrow R_f = [0, \infty)$$

$$\text{د) } y = (x^3 - 4x^2 + 3x + 1)^{\frac{1}{3}} \rightarrow R_f = \mathbb{R}$$

نارین زهر استغیث

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$$\text{الف) } y = \frac{x+1}{x-2}$$

$$R_f = \mathbb{R} - \{2\}$$

$$\text{ب) } y = \frac{x+1}{x-1}$$

$$R_f = \mathbb{R} - \{1\}$$

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