

$$f(x) = \sqrt{x + \sqrt{x-1}} \Rightarrow (\sqrt{x-1})^2 = x-1 \rightarrow x = (\sqrt{x-1})^2 + 1 \rightarrow x + \sqrt{x-1} = (\sqrt{x-1})^2 + 1 + \sqrt{x-1} \Rightarrow (\sqrt{x-1} + 1)^2$$

$$g(x) = \sqrt{x-1} \sqrt{x-1} \Rightarrow x-1 = (\sqrt{x-1} - 1)^2 \Rightarrow (\sqrt{x-1} - 1)^2$$

$$\rightarrow f(x) = \sqrt{(\sqrt{x-1} + 1)^2} \xrightarrow{x \geq 1} \sqrt{x-1} + 1 = f(x) \Rightarrow f(x) + g(x) = \sqrt{x-1} = a + b\sqrt{x-1} + c$$

$$\rightarrow g(x) = \sqrt{(\sqrt{x-1} - 1)^2} \xrightarrow{x \geq 1} \sqrt{x-1} - 1 = g(x)$$

$$a=0, b=1, c=-1$$

$$\frac{a+b}{c} = \frac{-1}{1} = -1$$

$$f(x) = \frac{x+1}{x^2-2x+3} \quad g(x) = \{(1,1), (1,0), (2,1), (0,1)\}$$

$$\hookrightarrow D_f = \mathbb{R} \setminus \{1, 2\} \quad \hookrightarrow D_g = \{(1,1), (1,0)\}$$

$$\frac{g}{f} \xrightarrow{x=1} \frac{1}{\frac{1}{1-2}} = -1 \Rightarrow \frac{g}{f} = \{(1, -1), (0, 1)\}$$

$$f(x) = \{(1,1), (1,-1), (2,1), (0,1)\} \quad g(x) = \{(1,1), (1,-1), (2,1), (0,1)\}$$

$$\hookrightarrow D_f = \{(1,1), (1,-1)\} \quad \hookrightarrow D_g = \{(1,1), (1,-1)\}$$

$$\text{مثال } f(x) = \begin{cases} x+1 \rightarrow x=0 \\ x-1 \rightarrow x=2 \\ x=1 \rightarrow x=1 \end{cases} \Rightarrow \{(1,1), (1,-1), (2,1), (0,1)\} \cdot f(x) = \begin{cases} x=1 \rightarrow x=1 \\ x=2 \rightarrow x=2 \\ x=1 \rightarrow x=1 \end{cases} \Rightarrow \{(1,1), (1,-1), (2,1), (0,1)\}$$

$$2) \frac{g}{f}(x) + 1 \Rightarrow \begin{cases} x=1 \rightarrow 1+1=2 \\ x=2 \rightarrow 2+1=3 \\ x=1 \rightarrow 1+1=2 \\ x=1 \rightarrow 1+1=2 \end{cases} \Rightarrow \{(1,2), (1,3), (2,2), (0,2)\} \cdot \frac{g}{f} = \begin{cases} x=1 \rightarrow \frac{1}{1-1} = -1 \\ x=2 \rightarrow \frac{1}{2-1} = 1 \\ x=1 \rightarrow \frac{1}{1-1} = -1 \end{cases} \Rightarrow \{(1,-1), (2,1), (1,-1)\}$$

$$f(x) = \{(1,1), (1,-1), (2,1), (0,1)\} \quad g(x) = \{(1,1), (1,-1), (2,1), (0,1)\}$$

$$\hookrightarrow D_f = \{(1,1), (1,-1)\} \quad \hookrightarrow D_g = \{(1,1), (1,-1)\}$$

$$h = f + g \Rightarrow \begin{cases} x=1 \rightarrow 1+1=2 \\ x=2 \rightarrow 2+1=3 \\ x=1 \rightarrow 1+1=2 \\ x=1 \rightarrow 1+1=2 \end{cases} \Rightarrow \{(1,2), (1,3), (2,2), (0,2)\}$$

$$f(x) = \sqrt{x^2 - x} \quad g(x) = [x] + [-x]$$

$$\hookrightarrow D_f = [0, 1] \setminus \{1\} \quad \hookrightarrow D_g = \mathbb{Z} \Rightarrow \{0, 1, 2\}$$

$$\Rightarrow \begin{cases} x=0 \rightarrow 0 \neq 1 \\ x=1 \rightarrow 1 \neq 1 \\ x=2 \rightarrow 2 \neq 1 \end{cases} \Rightarrow \{(1,1), (1,-1), (2,1)\}$$

$$\left(\frac{f}{g+1}\right)(x) = \frac{f(x)}{g(x)+1} \Rightarrow \begin{cases} x=0 \rightarrow \frac{0}{0+1} = 0 \\ x=1 \rightarrow \frac{1}{1+1} = \frac{1}{2} \\ x=2 \rightarrow \frac{2}{2+1} = \frac{2}{3} \end{cases} \Rightarrow \{(0,0), (1, \frac{1}{2}), (2, \frac{2}{3})\}$$

$$f(u) = \sqrt{4u - u^2 - 4}$$

$$4u - u^2 - 4 \geq 0$$

$$u^2 - 4u + 4 \leq 0$$

$$(u-1)(u-4) \leq 0$$

$$g(u) = \{(1, 2), (2, 2), (4, 2), (4, 1)\}$$

$$L Dg = \{(1, 2, 2), (2, 2, 2), (4, 2, 2), (4, 1, 2)\}$$

$$h(u) = \frac{f(u)}{g(u)} = \begin{cases} u=1 \rightarrow \frac{0}{2} = 0 \\ u=2 \rightarrow \frac{\sqrt{4}}{2} = 1 \\ u=4 \rightarrow \frac{0}{2} = 0 \end{cases} \Rightarrow \left\{ (1, 0), (2, 1), (4, 0) \right\}$$

6

$$\text{الف) } y = x^3 - 3x^2 + 3x \Rightarrow y = (x-1)^3 + 1 \rightarrow R_f = \mathbb{R}$$

$$\text{ب) } y = \frac{1}{x^2 - 2x} \Rightarrow x^2 y - 2xy = 1 \rightarrow y(x^2 - 2x) = 1 \rightarrow yx^2 - 2xy - 1 = 0 \xrightarrow{\Delta \geq 0} \begin{aligned} & \Delta y^2 + \Delta y \geq 0 \\ & \Delta y(y+1) \geq 0 \\ & \frac{-1 \pm 0}{2} = -\frac{1}{2} \end{aligned}$$

$$R_f = (-\infty, -\frac{1}{2}] \cup (0, \infty)$$

7

$$\text{الف) } y = x^2 - 4x + 1$$

$$\frac{-b}{2a} = \frac{4}{2} = 2 \rightarrow x = 2 \rightarrow y = -3 \rightarrow R_f = [-3, \infty)$$

$$\text{ب) } y = \sqrt{x^2 - 4x - 3}$$

$$\frac{-b}{2a} = \frac{2}{1} = 2 \rightarrow x = 2 \rightarrow y = -5 \rightarrow R_f = [-5, \infty)$$

$$\text{ب) } y = -x^2 + 4x + 3$$

$$\frac{-b}{2a} = \frac{-4}{-2} = 2 \rightarrow -4 + 16 + 3 = 15 \rightarrow R_f = (-\infty, 15]$$

$$\text{د) } y = \sqrt{4x - x^2}$$

$$\frac{-b}{2a} = \frac{-4}{-1} = 4 \rightarrow 16 - 16 = 0 \rightarrow [0, 4]$$

8

$$\text{الف) } y = x^3 - 4x^2 + 3x + 1 \rightarrow R_f = \mathbb{R}$$

$$\text{ب) } y = x^3 - 4x^2 + 3x - 2 \rightarrow R_f = \mathbb{R}$$

$$\text{ج) } y = \sqrt{x^3 - 4x^2 + 3x + 1} \rightarrow R_f = [0, \infty)$$

$$\text{د) } y = (x^3 - 4x^2 + 3x + 1)^{\frac{1}{2}} \xrightarrow{\text{مشتقات}} [0, \infty) = R_f$$

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9

$$\text{الف) } y = \frac{x+1}{x-2} \quad \text{جواب است}$$

$$R_f = \mathbb{R} - \left\{ \frac{1}{2} \right\} \rightarrow R_f = \mathbb{R} - \left\{ \frac{1}{2} \right\}$$

$$\text{ب) } y = \frac{x+1}{x+1} \quad \text{جواب است}$$

$$R_f = \mathbb{R} - \left\{ \frac{1}{2} \right\}$$

10