

حل المسألة

المسألة

$$\begin{aligned} f(x) &= \sqrt{x + f\sqrt{x-f}} \\ g(x) &= \sqrt{x - f\sqrt{x-f}} \end{aligned} \quad \left\{ \begin{array}{l} x > f \\ x = f \end{array} \right. \quad \begin{array}{l} \xrightarrow{x=f} \sqrt{f + f\sqrt{f-f}} = \sqrt{f} = f \\ \xrightarrow{x=f} \sqrt{f - f\sqrt{f-f}} = 0 \end{array} \quad \begin{array}{l} f+0=f \\ f+0=f \end{array}$$

$$\hookrightarrow \sqrt{x + f\sqrt{x-f}} - f + f + \sqrt{x - f\sqrt{x-f}} + f \Rightarrow \sqrt{(\sqrt{x-f} + f)^2} + \sqrt{(\sqrt{x-f} - f)^2} = 2$$

$$\left(\sqrt{x-f} + f + \sqrt{x-f} - f \right) \Rightarrow \frac{a}{b} \sqrt{\frac{c}{d}} = \frac{r+0}{-f} = \frac{-1}{f}$$

$$g(x) = \{(r, 1), (1, 0), (r, 0), (0, r)\} \quad f(x) = \frac{x+r}{x^2 - fx + r^2}$$

$$f(x) = \{(r, -r), (1, 0), (r, 0), (0, \frac{r}{r})\}$$

$$\frac{g}{f} \Rightarrow \{(r, \frac{1}{r}), (0, r)\}$$

$$\begin{aligned} g(x) &= (-r, r), (1, -1), (r, r), (-1, 0) \\ f(x) &= (1, r), (r, -1), (r, r), (-1, r) \end{aligned}$$

$$f(x) \Rightarrow \{(-\frac{1}{r}, r), (\frac{r}{r}, -1), (r, r), (r, r)\}$$

$$b) f(x^2) \Rightarrow \{(1, r), (\sqrt{r}, -1), (r, r), (\sqrt{-1}, r), (-r, r), (-\sqrt{r}, -1), (-1, r)\}$$

$$c) rg^r(x) + 1 \Rightarrow \{(-r, 1), (1, r), (r, r), (-1, 1)\}$$

$$d) \frac{rf}{g} \Rightarrow \{(1, -r), (r, -1), (-1, \frac{r}{r})\}$$

$$\begin{aligned} f(x) &= \{(r, 1), (r, 1), (r, r), (-r, 0)\} \\ g &= \{(0, 1), (r, r), (-r, r), (-1, r), (r, r)\} \end{aligned}$$

$$h = rf + g \rightarrow \{(r, 0), (r, 0), (-r, r)\}$$

$$\left(\frac{f}{g+1}\right)(x) \quad Dg = \mathbb{R} \quad g(x) = x(x-1) \quad f(x) = \frac{\sqrt{rx-x^2}}{x-1}$$

$$Df = (0, r) \cup (r, r)$$

$$x = \begin{cases} x \neq 1 \\ rx > 0 \\ x > 0 \end{cases}$$

$$\frac{x \neq 1}{rx > 0}$$

$$\sqrt{rx-x^2} - x \geq 0$$

$$\sqrt{rx-x^2} \geq x \rightarrow rx - x^2 \geq 0$$

$$x(r-x) \geq 0 \quad \begin{array}{c} 0 \quad r \Rightarrow [0, r] \\ - \quad + \quad - \end{array}$$

$$h(x) = \frac{f(x)}{g(x)}$$

$$g(x) = \{(1, r), (r, r), (r, r), (0, 1)\} \quad f(x) = \sqrt{0x - x^2 - f}$$

$$\hookrightarrow \{(1, \frac{r}{r}), (r, \frac{r}{r}), (r, \frac{r}{r})\}$$

$$f(x) = \{(1, 0), (r, \sqrt{r}), (r, 0), (0, \sqrt{-1})\}$$

$$\text{الف) } y = x^3 - 3x^2 + 3x \rightarrow y = (x-1)^3 + 1 \rightarrow y-1 = (x-1)^3 \quad (v)$$

$$\sqrt[3]{y-1} = x-1$$

$$x = \sqrt[3]{y-1} + 1 \rightarrow \mathbb{R}$$

$$\text{ب) } y = \frac{1}{x^2 - 2x} \Rightarrow x^2 - 2x = \frac{1}{y} \Rightarrow y(x^2 - 2x) = 1 \Rightarrow yx^2 - 2yx - 1 = 0$$

$$\Delta = 4y^2 + 4y \geq 0 \Rightarrow y(y+1) \geq 0 \Rightarrow y \leq -1 \text{ or } y \geq 0$$

$$\Rightarrow \mathcal{R} = (-\infty, -1] \cup [0, +\infty)$$

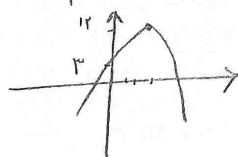
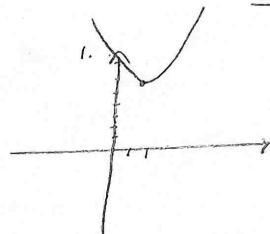
$$x = \frac{2y \pm \sqrt{4y^2 + 4y}}{2y} \quad y \neq 0$$

$$\text{الف) } y = x^2 - 2x + 1 \rightarrow \min \left| \frac{-b}{2a} = \frac{1}{1} = 1 \right|$$

$$\mathcal{R} = [1, +\infty)$$

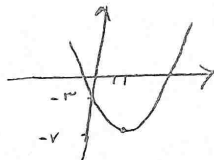
$$\text{ب) } y = -x^2 + 4x + 1 \rightarrow \max \left| \frac{-b}{2a} = \frac{-4}{-2} = 2 \right|$$

$$\mathcal{R} = (-\infty, 2]$$



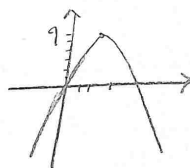
$$\text{ج) } y = \sqrt{x^2 - 4x + 4} \rightarrow \min \left| \frac{-b}{2a} = \frac{2}{1} = 2 \right|$$

$$\mathcal{R} = [2, +\infty)$$



$$\text{د) } y = \sqrt{4x - x^2} \Rightarrow \max \left| \frac{-b}{2a} = \frac{2}{-1} = -2 \right|$$

$$\mathcal{R} = [-2, 2]$$



$$\text{الف) } y = x^3 - 3x^2 + 3x + 1 \rightarrow \mathbb{R}$$

$$\text{ب) } y = x^3 - 4x^2 + 3x + 1 \rightarrow \mathbb{R}$$

$$\text{ج) } y = \sqrt{x^3 - 3x^2 + 3x + 1} \rightarrow \sqrt{\mathbb{R}} = [0, +\infty)$$

$$\text{د) } y = (x^3 - 3x^2 + 3x + 1)^2 \rightarrow \mathbb{R}^2 \rightarrow [0, +\infty)$$

$$\text{الف) } y = \frac{x^2 + 1}{x - 1} \rightarrow \mathbb{R} \setminus \{1\}$$

$$\text{ب) } y = \frac{x^2 + 1}{x + 1} \rightarrow \mathbb{R} \setminus \{-1\}$$