(۱)

$\frac{-b}{2a} = 2 \to \frac{-1}{a-1} = 2 \to -1 = 2a-2 \Rightarrow 2a = 1 \quad a = \frac{1}{2} \xrightarrow{a=\frac{1}{2}}$ (۱,۵)

$-\frac{1}{2}x^2 + x + 3 = 0 \xrightarrow{\Delta} 1 + 4 \times \frac{1}{2} \times 3 = 7 \to \frac{-1 \pm \sqrt{7}}{-1} \xrightarrow{\text{رشتی}} \frac{-1-\sqrt{7}}{-1}$

$x = 4, -2$ ✓ ✗

$f(x) = mx^2 - 5x + m$

(۱۲)

① $m < 0$

② $\Delta > 0 \to 25 - 4m^2 > 0 \to 4m^2 < 25 \to m^2 < \frac{25}{4}$ (۵)

$\xrightarrow{\text{اشتراک}} -\frac{5}{2} < m < 0$

$S = \frac{3-\sqrt{5}}{3} + \frac{3+\sqrt{5}}{3} = \frac{6}{3} = 2$

$P = \frac{3-\sqrt{5}}{3} \times \frac{3+\sqrt{5}}{3} = \frac{9-5}{9} = \frac{4}{9} \to x^2 - 2x + \frac{4}{9} = 0 \to 9x^2 - 18x + 4$ (۳)

(۵)

$3\alpha^2 + \beta^2 = 12 \to 2\alpha^2 + \alpha^2 + 2\beta^2 - \beta^2 = 12 \xrightarrow{\times -1} -2\alpha^2 - \alpha^2 - 2\beta^2 + \beta^2 = -12$

$-2(\alpha^2+\beta^2) + (\beta^2-\alpha^2) = -12 \to -2(14 - 2m - 4) + (\beta+\alpha)(\beta-\alpha)$

$\beta^2 - 4\beta$

$\frac{9\beta}{\alpha} = 4\beta \to 9\alpha^2 = 4\alpha^2 \to \alpha = \pm 3 \to \alpha = +3$

$S = 7 \to \alpha + \beta = 7 \to 3 + \beta = 7 \to \beta = 4$

$\frac{1}{3} + \frac{1}{4} = \frac{S}{P} = \frac{7}{12}$

(۱)

$\alpha + \beta = \frac{7}{4} \begin{cases} \alpha = 3 & \beta = \frac{1}{4} \times \\ \alpha = -3 & \beta = \frac{7}{4} \checkmark \end{cases}$

$\frac{1}{4} + \frac{1}{\beta} = \frac{7}{4}$

(۱,۵)

$ax^2 + bx + c = 0 \to ax^2 + bx + 20 \xrightarrow{\times 9} 9a + 2b + 5 = 9 \to 4\alpha + 2b = 4 \to 2\alpha + b = 2$ (۵)

$\frac{-b}{2a} = 2 \to -b = 4a \to b = -4\alpha \to -4\alpha + 2\alpha = 2 \to -2\alpha = 2$ (۵)

$-x^2 + 4x + 5 = 0 \to x^2 - 4x - 5 = 0 \to (x+1)(x-5) = 0$

$x \in (-1, 5)$

$\begin{array}{c|ccc} & -1 & & 5 \\ - & | & + & | & - \end{array}$

$\alpha < -1$

$b = 4$

(۱۴)

شهرزاد شوشتری

$\alpha^2 - m\beta = 8$ $\quad \alpha^2 = \alpha m + 2m \rightarrow -\alpha m - m\beta + 2m = 8 \rightarrow -m(\underbrace{\alpha+\beta}_{-m}) + 2m = 8 \rightarrow m^2 + 2m - 8 = 0 \rightarrow (m+4)(m-2) = 0 \;\; m = -4 \;\; m = 2$

-2 ↑ $-m$ $\quad$ (۱,۵) $\quad$ $D<0$ ✗ $\quad$ $D>0$ ✓

---

(۸) $f(x) = mx^2 + 4x + \frac{m}{2} + 7 \rightarrow \frac{-\Delta}{4a} = 8 \rightarrow \frac{-(16 - 4m(\frac{m}{2}+7))}{4m} = 8$

$\frac{-(16 - 2m^2 + 28m)}{4m} = 8 \rightarrow 32m = -16 + 2m^2 + 28m \rightarrow 2m^2 - 4m - 16 = 0$ (۵)

$m^2 - 2m - 8 = 0 \rightarrow (m-4)(m+2) = 0 \;\; m = \frac{8}{2} = 4 \;\; m = \frac{-4}{2} = -2$ ← بدون تابع ub max

$f(x) = -2x^2 + 4x + 4 \rightarrow x^2 + 2x - 8 = 0 \rightarrow (x+4)(x-2) = 0 \;\; x = \frac{-4}{2} = -2$ ✗ $\quad x = \frac{2}{-2} = -1$ ✓

$k = 2$

---

(۹) $ax^2 + bx + c = 0 \rightarrow ax^2 + bx + 4 = 0 \;\; (2,4) \;\; 4a + 2b + 4 = 0 \rightarrow 4a + 2b = -4$

$2a + b = 1 \quad \frac{-b}{2a} = 2 \rightarrow -b = 4a \quad b = -4a \quad 2a - 4a = 1 \rightarrow -2a = 1$ (۵)

$a = -\frac{1}{2} \rightarrow b = 2 \rightarrow -\frac{1}{2}x^2 + 2x + 4 = 0 \rightarrow \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta} = \frac{4}{-8} = \boxed{-\frac{1}{2}}$

---

(۱۰) $x^2 - (4a+4)x + (2a^2 + 4a + 3) = 0 \;\; \frac{x_1}{x_2} = 2 \rightarrow x_1 = 2x_2 \rightarrow 3x_2 = 4a + 4 \rightarrow x_2 = \frac{4a+4}{3}$

$x_1 = \frac{8a+8}{3} \rightarrow x_1 x_2 = 2a^2 + 4a + 3 \rightarrow \frac{(4a+4)(8a+8)}{9} = 2a^2 + 4a + 3$

$32a^2 + 64a + 32 = 18a^2 + 36a + 27 \rightarrow 14a^2 + 28a + 5 = 0 \rightarrow a = -1 \pm \frac{3}{14}$

$a = -\frac{1}{2} \quad a = -\frac{5}{2}$ $\quad$ (۱,۲۵)

$x^2 - 8x + 12 = 0 \rightarrow (x-2)(x-6) = 0 \rightarrow x = 2, 6$