(۱)

$\frac{-b}{2a} = 2 \to \frac{-1}{a-1} = 2 \to -1 = 2a-2 = 2a = 1 \quad a = \frac{1}{2} \xrightarrow{a=\frac{1}{2}}$

$-\frac{1}{2}x^2 + x + 3 = 0 \xrightarrow{\Delta} 1 + 4 \times \frac{1}{2} \times 3 = 7 \to \frac{-1 \pm \sqrt{7}}{-1} \xrightarrow{\text{ریشه‌ی +}} \frac{-1-\sqrt{7}}{-1}$

$f(x) = mx^2 - 5x + m$

① $m < 0$

② $\Delta > 0 \to 25 - 4m^2 > 0 \to 4m^2 < 25 \to m^2 < \frac{25}{4}$ (۱۲)

$\xrightarrow{\text{اشتراک}} -\frac{5}{2} < m < 0$

$S = \frac{3-\sqrt{5}}{3} + \frac{3+\sqrt{5}}{3} = \frac{6}{3} = 2$

$P = \frac{3-\sqrt{5}}{3} \times \frac{3+\sqrt{5}}{3} = \frac{9-5}{9} = \frac{4}{9}$ (۳)

$\to x^2 - 2x + \frac{4}{9} = 0 \to 9x^2 - 18x + 4$

$3\alpha^2 + \beta^2 = 12 \to 2\alpha^2 + \alpha^2 + 2\beta^2 - \beta^2 = 12 \xrightarrow{\times -1} -2\alpha^2 - \alpha^2 - 2\beta^2 + \beta^2 = -12$

$-2(\alpha^2+\beta^2) + (\beta^2-\alpha^2) = -12 \to -2(14 - 2m - 4) + (2)\frac{(9\xi - 8m - 14)}{2}$ (۴)

$5^2 - 4P$

$(\beta+\alpha)(\beta-\alpha)$

$ax^2+bx+c = 0 \to ax^2+bx+5 \xrightarrow{P=9} 4a + 2b + 5 = 9 \to 4\alpha + 2b = 4 \to 2\alpha + b = 2$ (۵)

$\frac{-b}{2a} = 2 \to -b = 4\alpha \to b = -4\alpha \to -4\alpha + 2\alpha = 2 \to -2\alpha = 2 \to \boxed{\alpha = -1}$

$-x^2 + 4x + 5 = 0 \quad x^2 - 4x - 5 = 0 \quad (x+1)(x-5) = 0 \quad \boxed{b = 4}$

$x \in (-1, 5)$

$P = \frac{9\beta}{\alpha} = 4\beta \to 9\alpha^2 \to \alpha^2 = 4 \to \alpha = \pm 2 \to \alpha = +2$

$S = 7 \to \alpha + \beta = 7 \to 2 + \beta = 7 \to \beta = 5$ (۶)

$\frac{1}{3} + \frac{1}{4} = \frac{5+3}{12} = \frac{7}{12}$

شهرزاد شوشتری

$\alpha^2 - m\beta = 8 \qquad \alpha^2 - \alpha m + 2m \to -\alpha m - m\beta + 2m = 8 \to -5 = -m$

$\to -m\underbrace{(\alpha+\beta)}_{-m} + 2m = 8 \to m^2 + 2m - 8 = 0 \to (m+4)(m-2) = 0 \quad m = -4 \quad m = 2$ (۷)

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(۸) $f(x) = mx^2 + 4x + \frac{m}{2} + 7 \to \frac{-\Delta}{4a} = 1 \to \frac{-(16 - 4m(\frac{m}{2}+7))}{4m} = 1$

$\frac{-(16 - 2m^2 + 28m)}{4m} = 1 \to 4m = -16 + 2m^2 + 28m \to 2m^2 - 24m - 16 = 0$

$m^2 - 12m - 32 = 0 \to (m-16)(m+2) = 0 \quad m = \frac{16}{2} = 8 \quad m = -\frac{2}{2} = -1$ چون تابع ub max

$f(x) = -2x^2 + 4x + 9 \to x^2 - 2x - 8 = 0 \quad (x-4)(x+2) = 0 \quad x = \frac{-4}{-2} = 2 \quad x = \frac{2}{-2} = -1$

$k = 2$

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(۹) $ax^2 + bx + c = 0 \to ax^2 + bx + 4 = 0 \xrightarrow{(2,4)} 4a + 2b + 4 = 0 \to 4a + 2b = 0$

$2a + b = 1 \qquad \frac{-b}{2a} = 2 \to -b = 4a \quad b = -4a \qquad 2a - 4a = 1 \to -2a = 1$

$a = -\frac{1}{2} \to b = 2 \to -\frac{1}{2}x^2 + 2x + 4 = 0 \to \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta} = \frac{4}{-8} = \boxed{-\frac{1}{2}}$

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(۱۰) $x^2 - (4a+4)x + (3a^2 + 9a + 3) = 0 \xrightarrow{x=1} 1 - (4a+4) + 3a^2 + 9a + 3 = 0$

$3a^2 + 5a = 0 \to a(3a+5) = 0 \quad a = 0, a = -\frac{5}{3}$

$x^2 - 8x + 12 = 0 \to (x-2)(x-6) = 0 \to x = 2, \boxed{x = 6}$