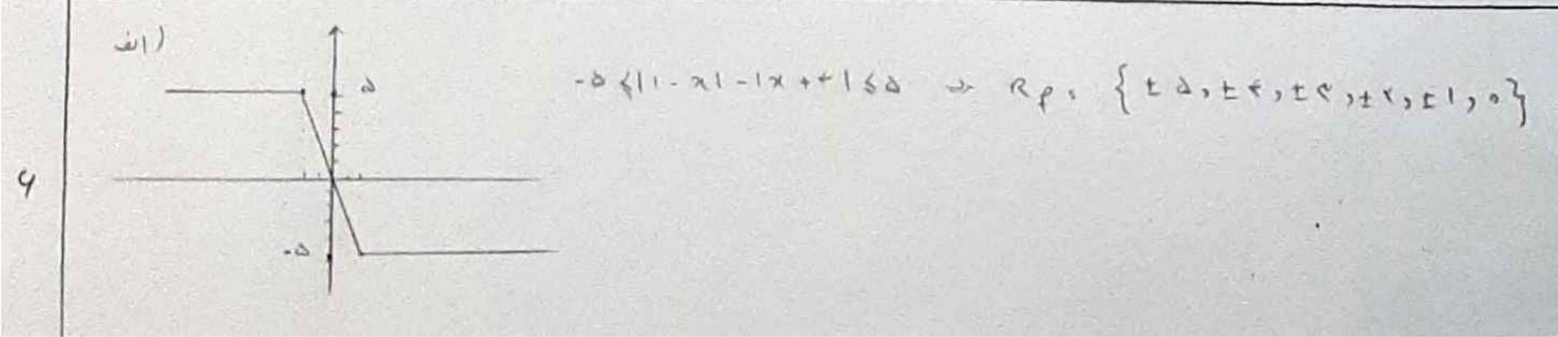
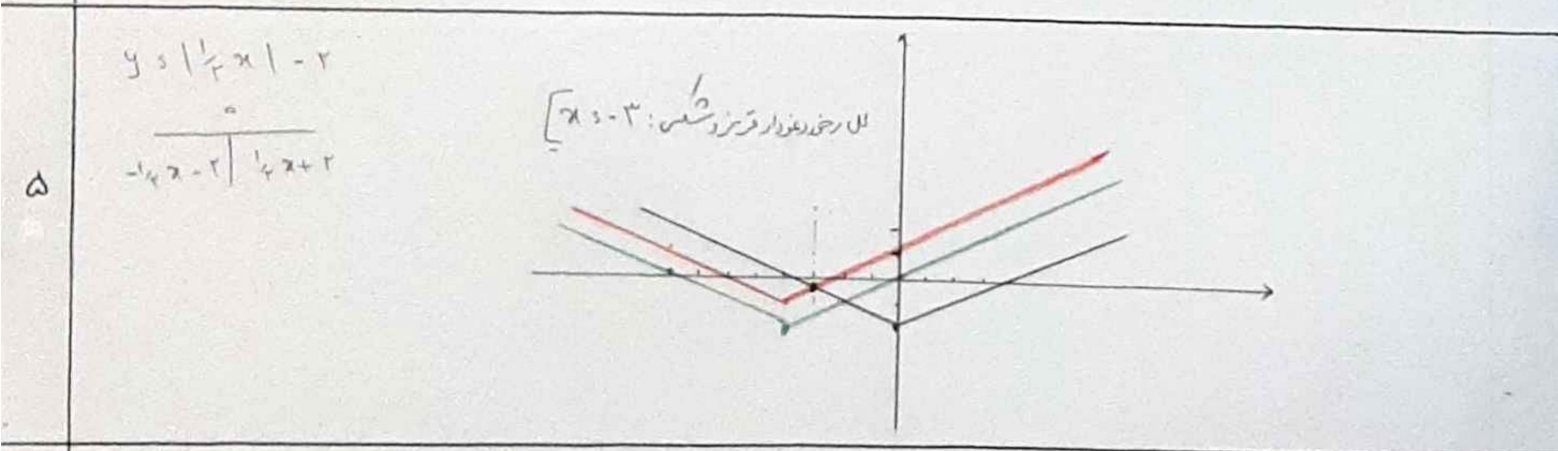
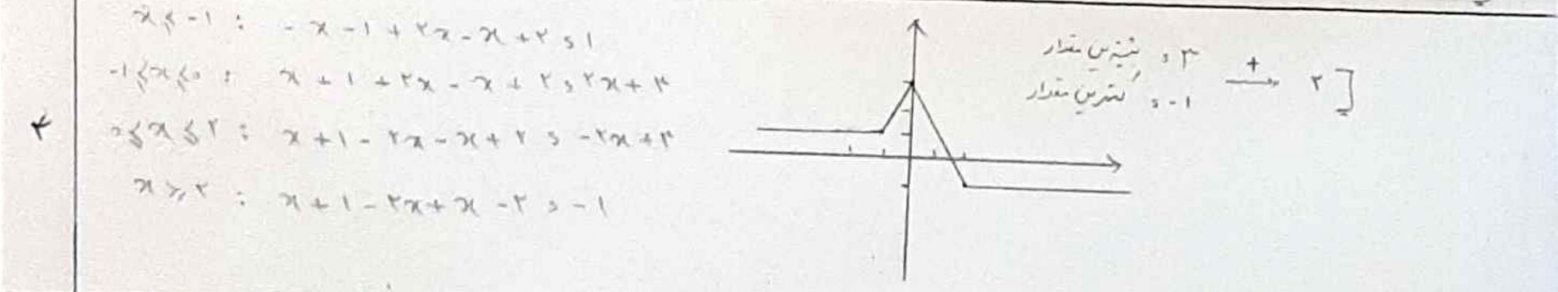


1 $\frac{1}{x-1} \leq x-1 \rightarrow (x-1)^2 \geq 1 \rightarrow x^2 - 2x + 1 \geq 1 \rightarrow x(x-2) \geq 0$
 $\rightarrow x \leq 0 \text{ یا } x \geq 2$
 بررسی: $x^2 - 1 \geq 1 \rightarrow x^2 \geq 2 \rightarrow x \leq -\sqrt{2} \text{ یا } x \geq \sqrt{2}$
 درستی است: $x=0 \rightarrow 1 \geq 1$ و $x=2 \rightarrow 1 \geq 1$

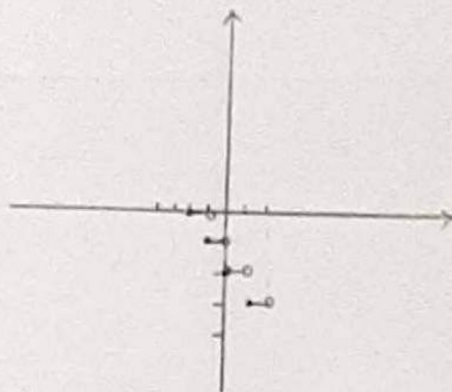
2 الف) $-2 \leq \frac{x-1}{x} < 2 \rightarrow \frac{x-1}{x} - 2 < 0 \rightarrow \frac{x-1-2x}{x} < 0 \rightarrow \frac{-x-1}{x} < 0$
 $\rightarrow \frac{x-1}{x} + 2 > 0 \rightarrow \frac{x-1+2x}{x} > 0 \rightarrow \frac{3x-1}{x} > 0$
 $\rightarrow x \in (-\infty, \frac{1}{3}) \cup (0, \infty)$
 ب) $\frac{x-2}{x+1} < -1 \rightarrow \frac{x-2+x+1}{x+1} < 0 \rightarrow \frac{2x-1}{x+1} < 0$
 $\frac{x-2}{x+1} > 1 \rightarrow \frac{x-2-x-1}{x+1} > 0 \rightarrow \frac{-x-3}{x+1} > 0$
 $\rightarrow x \in (-1, -3) \cup (-\infty, -1)$
 جواب نهایی: $\emptyset$

3 $x+4 > x^2 \rightarrow -x^2+x+4 > 0 \rightarrow x^2-x-4 < 0 \rightarrow (x-2)(x+2) < 0$
 $x+4 < -x^2 \rightarrow x^2+x+4 < 0 \rightarrow \Delta < 0$
 $\rightarrow |x-\alpha| < \beta \rightarrow -\beta < x-\alpha < \beta \rightarrow \alpha-\beta < x < \alpha+\beta$
 $\rightarrow \begin{cases} \alpha+\beta < 2 \\ \alpha-\beta < -2 \end{cases} \rightarrow \alpha < \frac{1}{2}$

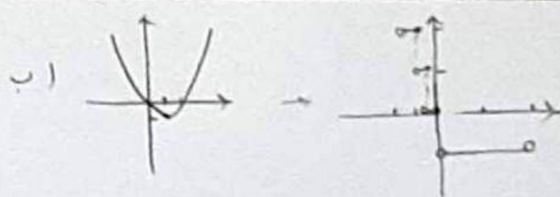
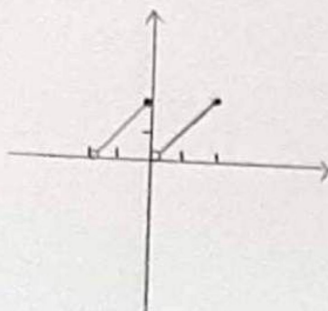


$$۲) \frac{1}{x} = \left[1 + \frac{1}{x} \right] \Rightarrow \frac{1}{x} = \underbrace{\left[\frac{1}{x} \right]}_t + 1 \Rightarrow 0 < t < 1 \Rightarrow 1 < t+1 < 2 \Rightarrow R_f \in (1, 2)$$

الف) $-1 < x \leq -\frac{1}{2} \Rightarrow 1 \leq -2x < 2 \xrightarrow{[0] \rightarrow 1} y \leq 1$
 $-\frac{1}{2} < x \leq 0 \Rightarrow 0 \leq -2x < 1 \xrightarrow{[0] \rightarrow 0} y \leq 0$
 $0 < x \leq \frac{1}{2} \Rightarrow -1 \leq -2x < 0 \xrightarrow{[-1] \rightarrow -1} y \leq -1$
 $\frac{1}{2} < x \leq 1 \Rightarrow -2 \leq -2x < -1 \xrightarrow{[-2] \rightarrow -1} y \leq -2$



ب) $x \in (-2, 2) \Rightarrow -1 < \frac{x}{2} < 1$
 $-1 < \frac{x}{2} \leq 0 \xrightarrow{\left[\frac{x}{2}\right] \rightarrow -1} y \leq x + 2 \quad (-2x \leq 0)$
 $0 < \frac{x}{2} < 1 \xrightarrow{\left[\frac{x}{2}\right] \rightarrow 0} y \leq x \quad (0 < x \leq 2)$



$a + [b] \in \mathbb{Z}, 2 \xrightarrow{b \in \mathbb{Z}} a, \text{ اعبار } 0, 2$
 معین : $[a] \in \mathbb{Z}, t-b \xrightarrow{[a] \in \mathbb{Z}} t, t-b \in \mathbb{Z} \Rightarrow \text{اعبار } b, 0, 2$
 $\Rightarrow \begin{cases} [a] + [b] \in \mathbb{Z} \\ [b] - [a] \in \mathbb{Z} \end{cases} \rightarrow \begin{cases} [b] \in \mathbb{Z} \Rightarrow b \in \mathbb{Z}, 2 \\ [a] \in \mathbb{Z} \Rightarrow a \in \mathbb{Z}, 2 \end{cases} \xrightarrow{+} \mathbb{Z}, 2$

$(1 - \sqrt{2}) < 1 \rightarrow$ با توان ۹ در سیر، باید عددهای در میان
 شود حد حتما
 $(1 + \sqrt{2})^4 \in 198 - (1 - \sqrt{2})^4 \Rightarrow [(1 + \sqrt{2})^9] \in 197$
 این عدد به صورت ۱۹۷ است
 (به طور تقریبی ۹۹۸)