

$\frac{1}{x-1} \geq |x-1|$

$\rightarrow \frac{1}{x-1} \geq 0 \rightarrow x-1 > 0 \rightarrow x > 1$

مقدار = ؟

① $\frac{1}{x-1} \geq x-1 \xrightarrow{(1-1)} \frac{1}{x-1} - (x-1) \geq 0 \xrightarrow{(1-2)} \frac{1-(x-1)^2}{x-1} \geq 0 \xrightarrow{(1-3)} \frac{1-x^2+2x-1}{x-1} \geq 0 \xrightarrow{(1-4)} \frac{-x^2+2x}{x-1} \geq 0 \xrightarrow{(1-5)} \frac{x(2-x)}{x-1} \geq 0$

② $\frac{1}{x-1} \geq 1-x \xrightarrow{(2-1)} \frac{1}{x-1} - (1-x) \geq 0 \xrightarrow{(2-2)} \frac{1-(1-x)^2}{x-1} \geq 0 \xrightarrow{(2-3)} \frac{1-1+2x-x^2}{x-1} \geq 0 \xrightarrow{(2-4)} \frac{-x^2+2x}{x-1} \geq 0 \xrightarrow{(2-5)} \frac{x(2-x)}{x-1} \geq 0$

الف) $\left| \frac{2x-1}{x} \right| < 2 \rightarrow -2 < \frac{2x-1}{x} < 2$

$\rightarrow \frac{2x-1}{x} - 2 < 0 \rightarrow \frac{2x-1-2x}{x} < 0 \rightarrow \frac{-1}{x} < 0 \rightarrow x > 0$

$\rightarrow 0 < \frac{2x-1}{x} - 2 \rightarrow \frac{2x-1-2x}{x} > 0 \rightarrow \frac{-1}{x} > 0 \rightarrow x < 0$

ب) $\left| \frac{x-2}{2x+1} \right| > 1 \rightarrow \frac{x-2}{2x+1} > 1 \rightarrow \frac{x-2-2x-1}{2x+1} > 0 \rightarrow \frac{-x-3}{2x+1} > 0$

$\frac{x-2}{2x+1} < -1 \rightarrow \frac{x-2}{2x+1} + 1 < 0 \rightarrow \frac{x-2+2x+1}{2x+1} < 0 \rightarrow \frac{3x-1}{2x+1} < 0$

$x < |x+y| \rightarrow |x-\alpha| < \beta$

$\rightarrow -\beta < x-\alpha < \beta \rightarrow \alpha-\beta < x < \alpha+\beta$

$\alpha = ?$

$\rightarrow -x < x^2 < x+y \rightarrow x^2 - x - y < 0 \rightarrow (x^2 - x - y) < 0$

$\rightarrow 0 < x^2 - x - y \rightarrow x^2 - x - y > 0$

$y \geq |x+1| - |2x| + |x-2| \rightarrow$

① $y \geq 1 \rightarrow$

② $y \geq -1 \rightarrow$

③ $y \geq 1 \rightarrow$

④ $y \geq -1 \rightarrow$

$y \geq \left| \frac{1}{x} \right| - 2$

$\rightarrow \left| \frac{1}{x} \right| - 2 \geq 0 \rightarrow \left| \frac{1}{x} \right| \geq 2 \rightarrow \frac{1}{x} \geq 2 \text{ or } \frac{1}{x} \leq -2$

$\rightarrow \frac{1}{x} \geq 2 \rightarrow x \leq \frac{1}{2}$

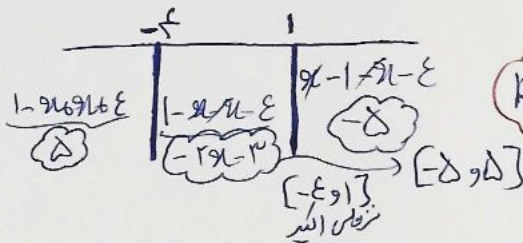
$\rightarrow \frac{1}{x} \leq -2 \rightarrow x \geq -\frac{1}{2}$

$\left| \frac{1}{x} \right| - 2 \geq 0 \rightarrow \left| \frac{1}{x} \right| \geq 2$

$\rightarrow \frac{1}{x} \geq 2 \text{ or } \frac{1}{x} \leq -2$

$\rightarrow x \leq \frac{1}{2} \text{ or } x \geq -\frac{1}{2}$

الف) $f(x) = [1-x-|x+\epsilon|] \rightarrow R_f = ?$



$R_f = \{x \mid x \in \mathbb{R}, -1-\epsilon \leq x \leq -\epsilon\}$

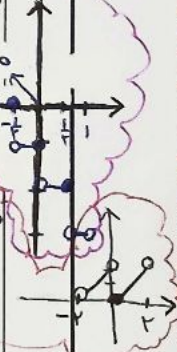
ب) $f(x) = \frac{1}{x} - \left\lfloor \frac{x+1}{x} \right\rfloor$

$R_f = \mathbb{R} \quad \frac{1}{x} - \left\lfloor 1 + \frac{1}{x} \right\rfloor \rightarrow \frac{1}{x} - \left\lfloor \frac{1}{x} \right\rfloor - 1 \quad R_f = [-1, 0)$

الف) $y = [-2x] - 1$

$x \in (-1, 1) \rightarrow$

$-1 < x < 0 \rightarrow [-2x] = 0 \rightarrow y = -1$
 $0 \leq x < 1 \rightarrow [-2x] = -1 \rightarrow y = -2$

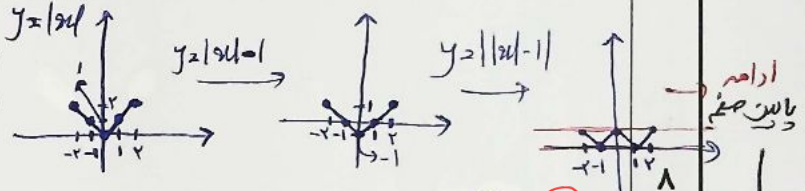


ب) $y = x - 2 \left\lfloor \frac{x}{2} \right\rfloor$ $x \in (-2, 2)$

$-2 < x < -1 \rightarrow \left\lfloor \frac{x}{2} \right\rfloor = -1 \rightarrow y = x + 2$
 $-1 \leq x < 0 \rightarrow \left\lfloor \frac{x}{2} \right\rfloor = 0 \rightarrow y = x$
 $0 \leq x < 1 \rightarrow \left\lfloor \frac{x}{2} \right\rfloor = 0 \rightarrow y = x$
 $1 \leq x < 2 \rightarrow \left\lfloor \frac{x}{2} \right\rfloor = 1 \rightarrow y = x - 2$

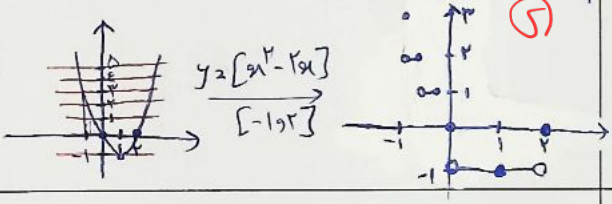
الف) $y = [1-x-1]$

$x \in [-2, 2]$



ب) $y = [x^2 - 2x]$

$x \in [-1, 2]$



$b - [a] \in \mathbb{Z}, \epsilon$

$[b] - [a] \in \mathbb{Z}$

$a \in [b] \in \mathbb{Z}, \epsilon$

$[a] + [b] \in \mathbb{Z}$

$2[b] \geq 4$
 $[b] \geq 2, [a] \geq 1$
 $b - [a] \in \mathbb{Z}, \epsilon$

$a + b \in \mathbb{Z}$

$a + b \in \mathbb{Z}, \epsilon$

$[b] \geq 2, \epsilon$
 $a + [b] \in \mathbb{Z} \rightarrow [a] \geq 1, \epsilon$

$(1+\sqrt{2})^7 + (-1-\sqrt{2})^7 = 198$

$\rightarrow (1+\sqrt{2})^7 = 198 - (-1-\sqrt{2})^7$

$\{ (1+\sqrt{2})^7 \} = ? \rightarrow \{ (1+\sqrt{2})^7 \} = \{ 198 - (-1-\sqrt{2})^7 \}$

$+ 2198 + [- (-1-\sqrt{2})^7]$

$+ 2198 - 1$

$1 < \sqrt{2} < 2 \rightarrow -2 < -\sqrt{2} < -1 \rightarrow -3 < -1-\sqrt{2} < -2$
 $0 < (1+\sqrt{2})^7 < 1 \rightarrow 0 < (-1-\sqrt{2})^7 < -1$
 $\rightarrow [(-1-\sqrt{2})^7] = -1$

