حل معادله

۱- چند ریشه؟ ~~۲ ریشه~~ یک ریشه — ۱٫۵

$\frac{1}{x-1} = |x-1| \to 1 < x \qquad \frac{1}{x-1} = x-1 \to (x-1)^2 = 1$ — ۱٫۵

$x-1 = \pm 1 \to x = 2$ ✓

$x = 0$ ✗ غ ق ق

$x < 1 \to \frac{1}{x-1} = -(x-1) \to -(x-1)^2 = 1 \to$ غ ق ق

۲- الف) $\left|\frac{2x-1}{x}\right| < 3$ — ۱٫۵

$-3 < \frac{2x-1}{x} < 3 \to -3 < +2 - \frac{1}{x} < 3 \xrightarrow{-2} -5 < -\frac{1}{x} < 0 \xrightarrow{\text{قرینه و معکوس}} \frac{1}{5} < x < +\infty$

$D_x = (\frac{1}{5}, \infty)$ ✓

ب) $\left|\frac{x-2}{2x+1}\right| > 1$

$|x-2|^2 > |2x+1|^2 \to 3x^2 + 8x - 3 < 0 \to -3 < x < \frac{1}{3}, \; x \neq -\frac{1}{2}$

$1 < \frac{x-2}{2x+1} \to 0 < \frac{x-2}{2x+1} - 1 \Rightarrow 0 < \frac{-x-3}{2x+1}$ ($-(x+3)$)

$-3, \; -\frac{1}{2}$: $- \; + \; -$ $\qquad x = (-3, -\frac{1}{2})$

$-\left(\frac{x-2}{2x+1}\right) < 1 \to$

۳- $x^2 < |x+6| \to |x-\alpha| < \beta \qquad \alpha = ?$ — ۲

$-(x+6) < x^2 < x+6$

$x^2 - x - 6 < 0$ $\qquad -2, \; 3$: $+ \; - \; +$

$x^2 + x + 6 < 0 \to \Delta < 0$ $\qquad |x - \frac{1}{2}| < \frac{5}{2} \to \alpha = \frac{1}{2} \qquad \beta = \frac{5}{2}$

۴- $y = |x+1| - |2x| + |x-2|$ $\qquad max + min = ?$ — ۲

$-1, \; 0, \; 2$

$y = 1 \qquad y = 2x+3 \qquad y = x+1-2x-2x \qquad -1 = y$

$max = 3 \qquad 3 - 1 = 2$

$min = -1$

$y = |x+1| - |2x| + |x-2|$

-۴ max + min?

Number line: -1, 0, 2

$y=1$ $y = 2x+3$ $y = x+1-2x-2x$ $-1 = y$

$y = -2x+3$

$\begin{bmatrix}-1\\1\end{bmatrix}$ $\begin{bmatrix}0\\3\end{bmatrix}$ $\begin{bmatrix}2\\-1\end{bmatrix}$

$max = 4$ $4-1=3$

$min = -1$

حلیا صفدریان

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$y = |\frac{1}{2}x| - 2$ $\rightarrow x' = x+4$ $y' = y+1$ $(0,-2)$

نمودار اولیه و ثانویه در کدام نقطه تقاطع؟

$y = |\frac{1}{2}(x+4)| - 2 + 1 \Rightarrow y = |\frac{1}{2}(x+4)| - 1$

به توان ۲

$|\frac{1}{2}(x+4) - 1| - 1 = |\frac{1}{2}x| - 2 \rightarrow |\frac{1}{2}x + 1| = |\frac{1}{2}x| - 1 \rightarrow t^2 + 2t + 1 = t^2 + 1 - 2t$

$\frac{1}{2}x + \frac{2}{2} - 1$

$4t = 0 \rightarrow t = 0$

$\frac{1}{2}x = 0 \rightarrow x = 0$ $y = -2$

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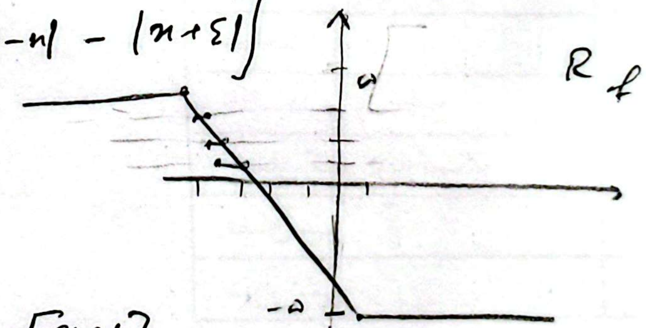
-۶ الف) $f(x) = [\,|1-x| - |x+4|\,]$

$|1-x| - |x+4|$



$R_f = [5, 4, 3, 2, 1, 0, \ldots, -4, -5]$ ‍ بررسی؟

$R_f = [-1, 3)$

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ب) $f(x) = \frac{1}{x} - [\frac{x+1}{x}]$

$1 + \frac{1}{x}$

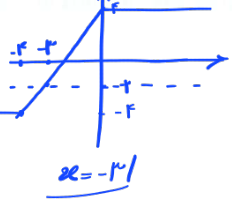
$f(x) = \frac{1}{x} - 1 - [\frac{1}{x}] = \frac{1}{x} - [\frac{1}{x}] - 1$

$R_{(t-[t])} = [0, 1)$ $\xrightarrow{t-[t]-1}$ ~~$[-1, -2)$~~ $[-1, 0)$

$t - [t] = 0$ $t \in Z$ $\longrightarrow f(x) = -1$

$y' = |\frac{1}{2}x+2|-1 = y$ (۵

$|x+4|-|x|=-2$



۷- الف) رسم نمودار

الف) $y=[-2x]-1$ $x\in(-1,1)$

$x \neq \frac{1}{2}$ $-1<x<-\frac{1}{2} \Rightarrow [-2x]-1=1$ $\frac{1}{2}<x<1$ $y=-2$

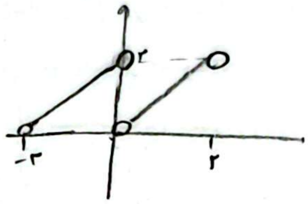
$-\frac{1}{2}<x<0 \Rightarrow [-2x]-1=0$

$0<x<\frac{1}{2} \Rightarrow [-2x]-1=0 \to y=-1$

ب) $y=x-2[\frac{x}{2}]$ $x\in(-2,2)$

$k=2x$ $-2<x<0 \to y=x+2$

$0\le x<2 \to y=x$



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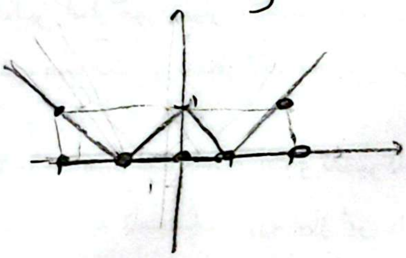
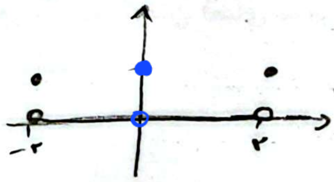
۸- رسم نمودار

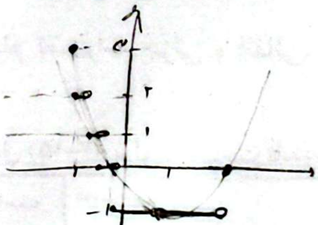
الف) $y=[||x|-1|]$ $x\in[-2,2]$





ب) $y=[x^2-2x]$ $x\in[-1,2]$

$y=[\underbrace{x^2-2x+1}_{(x-1)^2}-1]$



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$a+[b]=4/2$ $a+b=?$

$b-[a]=3/4$

$a+b=1/2+3/4=4/6$

$a=1/2$

$b=3/4$

$[a]+[b]=4$

$[b]-[a]=2$

$2[b]=6 \to [b]=3 \to [a]=1$

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۱۰- جزء عدد صحیح $P=(1+\sqrt{2})^4$

$(1+\sqrt{2})^4+(1-\sqrt{2})^4=198$

$0<(1-\sqrt{2})^4<1$ $[(1+\sqrt{2})^4]=[198-(1-\sqrt{2})^4]$

$=[(1+\sqrt{2})^4]=198+[\underbrace{-(1-\sqrt{2})^4}_{0^-}]=198-1=197$

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