حل معادلات

۱- چند ریشه؟ ۲ ریشه

$\frac{1}{x-1}=|x-1| \to 1<x \quad \frac{1}{x-1}=x-1 \to (x-1)^2=1$

$x-1=\pm1 \to x=2$
$x=0$

$x<1 \quad \frac{1}{x-1}=-(x-1) \to -(x-1)^2=1 \to$ غ‌ق

۲- الف) $\left|\frac{2x-1}{x}\right|<2$

$-2<\frac{2x-1}{x}<2 \to -2<+2-\frac{1}{x}<2 \xrightarrow{-2} -4<-\frac{1}{x}<0 \xrightarrow{\text{قرینه و معکوس}} \frac{1}{4}<x<+\infty$

$D_x=(\frac{1}{4},+\infty)$

ب) $\left|\frac{x-2}{2x+1}\right|>1$

$1<\frac{x-2}{2x+1} \to 0<\frac{x-2}{2x+1}-1 \Rightarrow 0<\frac{\overbrace{-x-3}^{-(x+3)}}{2x+1}$ $\quad$ $-3 \quad -\frac{1}{2}$: $- \quad + \quad -$ $\quad x=(-3,-\frac{1}{2})$

$-\left(\frac{x-2}{2x+1}\right)<1 \to$

۳- $x^2<|x+6| \to |x-\alpha|<\beta \qquad \alpha=?$

$\underbrace{-(x+6)<x^2}<x+6$

$x^2-x-6<0 \quad$ $-2 \quad 3$: $+ \quad - \quad +$

$x^2+x+6<0 \to \Delta<0 \qquad |x-\frac{1}{2}|<\frac{5}{2} \to \alpha=\frac{1}{2} \qquad \beta=\frac{5}{2}$

۴- $y=|x+1|-|2x|+|x-2| \qquad \max+\min=?$

$-1 \qquad 0 \qquad 2$

$y=1 \quad y=2x+3 \quad y=x+1-2x-2x \quad -1=y$

$\max=3 \qquad 3-1=2$

$\min=-1$

$y = |x+1| - |2x| + |x-2|$

max , min ? -4

-1 $\qquad 0 \qquad 2$

$y=1 \qquad y=2x+4 \qquad y=x+1-2x-2x \qquad -1=y$

$y=-2x-3$

$\max = 3 \qquad 3-1=2$

$\min = -1$

$\begin{bmatrix}-1\\1\end{bmatrix} \qquad \begin{bmatrix}0\\3\end{bmatrix} \qquad \begin{bmatrix}2\\-1\end{bmatrix}$

حل با صفریان

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$y=|\frac{1}{2}x|-2 \qquad \to x'=x+4 \qquad y'=y+1 \qquad (0,-2)$

نمودار اولیه و ثانویه در کدام نقطه متقاطع؟

$y=|\frac{1}{2}(x+4)|-2+1 \Rightarrow y=|\frac{1}{2}(x+4)|-1$

به توان ۲

$|\frac{1}{2}(x+4)-1|-1=|\frac{1}{2}x|-2 \to |\underbrace{\frac{1}{2}x}_{t}+1| = |\underbrace{\frac{1}{2}x}_{t}|-1 \to t^2+2t+1 = t^2+1-2t$

$\frac{1}{2}x+\cancel{2}-1$

$4t=0 \to t=0$

$\frac{1}{2}x=0 \to \boxed{x=0} \qquad \boxed{y=-2}$

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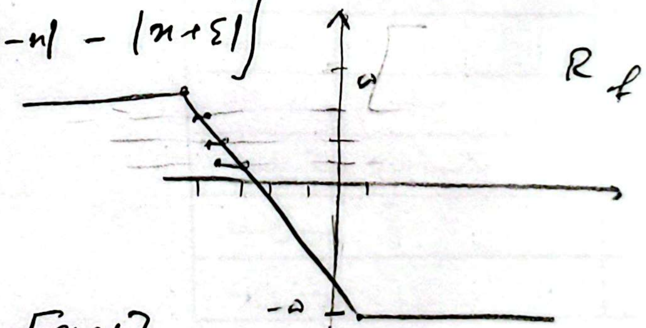
الف) $f(x)=[|1-x|-|x+4|]$

$|1-x|-|x+4|$



$R_f=[5,4,3,2,1,0,\dots,\overset{\text{درسته؟}}{-4},-5]$

$R_f=[-1,2)$

ب) $f(x)=\frac{1}{x}-[\frac{x+1}{x}]$

$1+\frac{1}{x}$

$f(x)=\frac{1}{x}-1-[\frac{1}{x}] = \underbrace{\frac{1}{x}}_{t}-[\underbrace{\frac{1}{x}}_{t}]-1$

$R_{(t-[t])}=[0,-1) \xrightarrow{t-[t]-1} [-1,-2)$ ؟

$t-[t]=0 \qquad t\in Z \longrightarrow f(x)=-1$

حل‌المسائل

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الف) رسم توابع

الف) $y=[-2x]-1$ $\qquad x\in(-1,1)$

$k=\frac{1}{2}$ $\qquad -1<x<-\frac{1}{2} \Rightarrow [-2x]-1=1$ $\qquad \frac{1}{2}<x<1$ $\qquad y=-3$

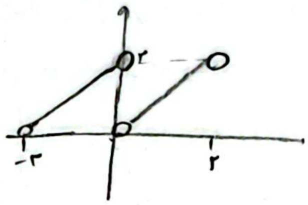
$-\frac{1}{2}\le x<0 \Rightarrow [-2x]-1=0$

$0\le x<\frac{1}{2} \Rightarrow [-2x]-1=0 \to y=-1$

ب) $y=x-2[\frac{x}{2}]$ $\qquad x\in(-2,2)$

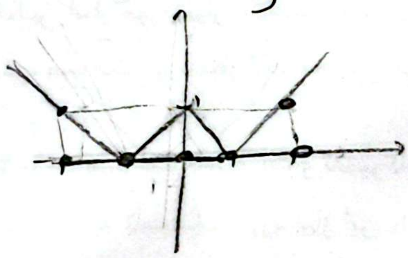
$k=2x$ $\qquad -2<x<0 \to y=x+2$

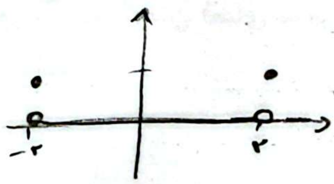
$0\le x<2 \to y=x$



-۸ رسم نمودار

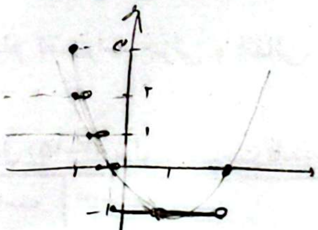
الف) $y=[||x|-1|]$ $\qquad x\in[-2,2]$





ب) $y=[x^2-2x]$ $\qquad x\in[-1,2]$

$y=[\underbrace{x^2-2x+1}_{(x-1)^2}-1]$



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$a+[b]=4.2$ $\qquad a+b=?$

$b-[a]=2.4$ $\qquad a+b=1.2+3.4=4.6$

$a=1.2$

$b=3.4$

$[a]+[b]=4$

$[b]-[a]=2$

$2[b]=6 \to [b]=3 \to [a]=1$

-۱۰ جزء عدد صحیح $P=(1+\sqrt{2})^6$?

$(1+\sqrt{2})^6+(1-\sqrt{2})^6=198$