باران کاظمی تکلیف شماره ۲۰ یازدهم C

۱۹٫۵

۱)
$$\frac{1}{x-1} = |x-1| \begin{cases} x-1 = \frac{1}{x-1} \xrightarrow{x \neq 1} (x-1)^2 = 1 \to x^2 - 2x + \cancel{1} = \cancel{1} \to x(x-2) = 0 \to x = 0 \text{ غ‌ق‌ق},\ \boxed{x = 2} \\ x-1 = -\frac{1}{x-1} \to x^2 - 2x + 1 = -1 \to x^2 - 2x + 2 = 0 \end{cases}$$

$$\frac{1}{x-1} \geq 0 \quad \frac{1}{-\ 1\ +} \to (1, +\infty)$$

$$\frac{2 \pm \sqrt{4-8}}{2} \to \Delta < 0 \quad \text{غ‌ق‌ق}$$

یک جواب دارد

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۲) الف)
$$\left|\frac{2x-1}{x}\right| < 2 \to -2 < \frac{2x-1}{x} < 2 \begin{cases} \frac{2x-1}{x} - 2 < 0 \to \frac{\cancel{2x}-1-\cancel{2x}}{x} < 0 \quad \frac{0}{+\ \ -} \quad (0, +\infty) \\ 0 < \frac{2x-1}{x} + 2 \to 0 < \frac{2x-1+2x}{x} \to 0 < \frac{4x-1}{x} \end{cases}$$

$$\frac{0 \quad \frac{1}{4}}{+ \mid - \mid +} \to (-\infty, 0) \cup (\tfrac{1}{4}, +\infty)$$

$$\xrightarrow{\cap} (\tfrac{1}{4}, +\infty)$$

ب)
$$\left|\frac{x-2}{2x+1}\right| > 1 \to -1 > \frac{x-2}{2x+1} > 1 \begin{cases} \frac{x-2-2x-1}{2x+1} > 0 \to \frac{-x-3}{2x+1} > 0 \quad \frac{-3 \quad -\frac{1}{2}}{- \mid + \mid -} \quad (-3, -\tfrac{1}{2}) \\ \frac{x-2}{2x+1} < -1 \to \frac{x-2+2x+1}{2x+1} < 0 \to \frac{3x-1}{2x+1} < 0 \quad \frac{-\frac{1}{2} \quad \frac{1}{3}}{+ \mid - \mid +} \quad (-\tfrac{1}{2}, \tfrac{1}{3}) \end{cases}$$

$$\longrightarrow (-3, \tfrac{1}{3}) - \{\tfrac{1}{2}\}$$

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۳)
$$x^2 < |x+6| \to x^2 < x+6 < -x^2 \begin{cases} x^2 - x - 6 < 0 \quad (x-3)(x+2) < 0 \quad \frac{-2 \quad 3}{+ \mid - \mid +} \quad (-2, 3) \\ x^2 + x + 6 < 0 \quad 1 - 24 < 0 \quad \Delta < 0 \quad \text{غ‌ق‌ق} \end{cases}$$

$$-2 < x < 3$$
$$|x - \tfrac{1}{2}| < \tfrac{5}{2}$$
$$\boxed{\alpha = \tfrac{1}{2}}$$

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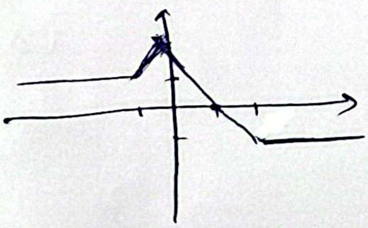
۴)
$$y = |x+1| - |2x| + |x-2|$$

	-1	0	2	
$-\cancel{x}-1+2x-\cancel{x}+2$	$x+1+2x-x+2$		$x+1-2x+x-2$	
1	$2x+3$	$x+1-2x-x+2$	-1	
		$-2x+3$		



$$\left.\begin{array}{l} \text{کمترین} = -1 \\ \text{بیشترین} = 3 \end{array}\right\} \to 3 - 1 = \boxed{2}$$

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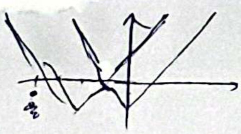
۵)
$$y = |\tfrac{1}{2}x| - 2 \qquad y' = |\tfrac{1}{2}x + 2| - 1$$

$$|\tfrac{1}{2}x| - 2 = |\tfrac{1}{2}x + 2| - 1$$

$$-\tfrac{1}{2}x - 2 = \tfrac{1}{2}x + \cancel{2} - 1 \qquad x = -3$$

$$-3, \quad -\tfrac{1}{2}$$

$$\frac{-4}{-\frac{1}{2}x - 3 \quad \frac{1}{2}x + 1}$$



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۶)

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الف) $f(x) = [|1-x| - |x+4|]$

$|1-x| - |x+4|$

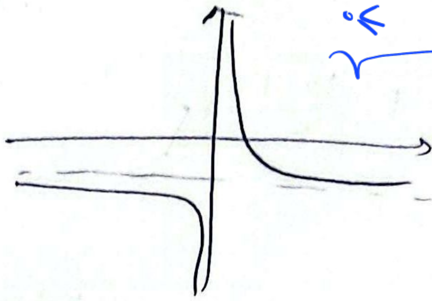
-4		$+1$	
$1-x+x+4 = 5$	$1-x-x-4$ / $-2x-3$		$x-1-x-4 = -5$

$R_f = \{-5, -4, \dots, 4, 5\}$

ب) $f(x) = \frac{1}{x} - \left[\frac{x+1}{x}\right]$, $\frac{1}{x} - \left[1 + \frac{1}{x}\right]$, $\underbrace{\frac{1}{x} - \left[\frac{1}{x}\right]}_{0 \leq \quad < 1} - 1$ $\rightarrow R_f = [-1, 0)$

$f(x) = \frac{1}{x} - 1$

$R_f = (-\infty, -1) \cup (-1, +\infty)$



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الف) $y = [-2x] - 1 \qquad x \in (-1, 1)$

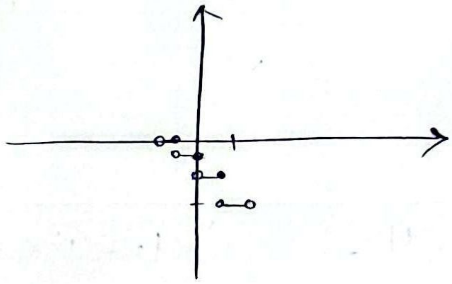
$-1 < x < 1 \rightarrow 2 > -2x > -2 \rightarrow -2 < -2x < 2$

$-2 < -2x < -1 \xrightarrow{[-2x] = -2} -3 \qquad \frac{1}{2} < x < 1$

$-1 \leq -2x < 0 \xrightarrow{[-2x] = -1} -2 \qquad 0 < x \leq \frac{1}{2}$

$0 \leq -2x < 1 \xrightarrow{[-2x] = 0} -1 \qquad -\frac{1}{2} < x \leq 0$

$1 \leq -2x < 2 \xrightarrow{[-2x] = 1} 0 \qquad -1 < x \leq -\frac{1}{2}$



ب) $y = x - 2\left[\frac{x}{2}\right] \qquad x \in (-2, 2)$

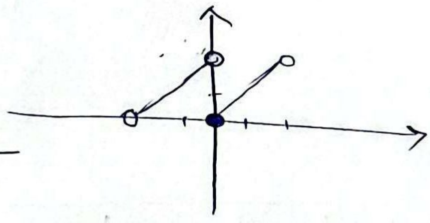
$-2 < x < 2 \rightarrow -1 < \frac{x}{2} < 1$

$-1 < \frac{x}{2} < 0 \xrightarrow{[\frac{x}{2}] = -1} x + 2 \qquad -2 < x < 0$

x	y
-2	0
0	2

$0 \leq \frac{x}{2} < 1 \xrightarrow{[\frac{x}{2}] = 0} x \qquad 0 \leq x < 2$

x	y
0	0
2	2



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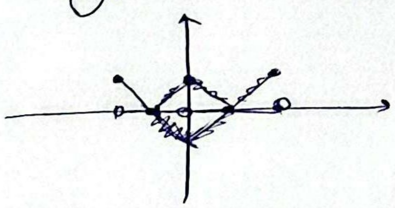
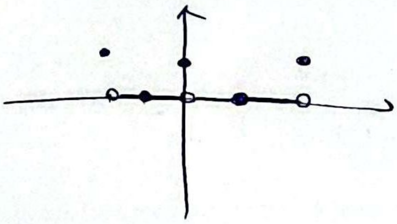
۸)

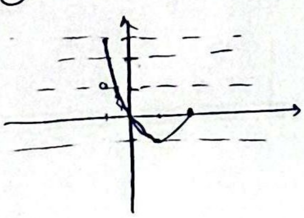
الف) $y = [||x| - 1|] \qquad x \in [-2, 2]$



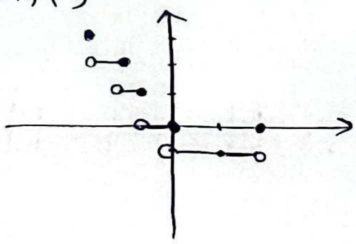
$\rightarrow$



ب) $y = [x^2 - 2x] \qquad x \in [-1, 2]$



$\rightarrow$



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$a + [b] = ۴,۲ \rightarrow \boxed{a = ۱,۲}$

$[a + [b]] = [۴,۲]$

$\begin{cases} [a] + [b] = ۴ \\ [b] - [a] = ۲ \end{cases} \rightarrow [a] = ۱$

$۲[b] = ۶ \rightarrow [b] = ۳$

$b - [a] = ۲,۴ \longrightarrow \boxed{b = ۳,۴}$

$[b - [a]] = [۲,۴]$

$[b] - [a] = ۲$

$a + b = ۱,۲ + ۳,۴ = \boxed{۴,۶}$

(۱۰

$(۱+\sqrt{۲})^۶ + (۱-\sqrt{۲})^۶ = ۱۹۸$

$۱ - \sqrt{۲} = ۱ - ۱,۴ = -۰,۴ \rightarrow -۱ < ۱ - \sqrt{۲} < ۰ \rightarrow ۰ < (۱-\sqrt{۲})^۶ < ۱$

$\underset{۱۹۷}{۱۹۸-۱} < (۱+\sqrt{۲})^۶ < ۱۹۸ \rightarrow [(۱+\sqrt{۲})^۶] = \boxed{۱۹۷}$