باران کاظمی تکلیف شماره ۲۰ یازدهم C

۱)
$$\frac{1}{x-1}=|x-1| \begin{cases} x-1=\frac{1}{x-1} \xrightarrow{x\neq 1} (x-1)^2=1 \rightarrow x^2-2x+\cancel{1}=\cancel{1} \rightarrow x(x-2)=0 \begin{cases} x=0 \text{ غ ق ق} \\ \boxed{x=2} \end{cases} \\ x-1=-\frac{1}{x-1} \rightarrow x^2-2x+1=-1 \rightarrow x^2-2x+2=0 \end{cases}$$

$$\frac{1}{x-1}\geq 0 \quad \frac{1}{-\;+} \rightarrow (1,+\infty)$$

$$\frac{2\pm\sqrt{4-8}}{2} \rightarrow \Delta<0 \text{ غ ق ق}$$

یک جواب دارد

۲) الف)
$$\left|\frac{2x-1}{x}\right|<2 \rightarrow -2<\frac{2x-1}{x}<2 \begin{cases} \frac{2x-1}{x}-2<0 \rightarrow \frac{\cancel{2x}-1-\cancel{2x}}{x}<0 \quad \frac{0}{+\;-} \quad (0,+\infty) \\ 0<\frac{2x-1}{x}+2 \rightarrow 0<\frac{2x-1+2x}{x} \rightarrow 0<\frac{4x-1}{x} \end{cases}$$

$$\begin{array}{c|c|c|c} & 0 & \frac{1}{4} & \\ + & - & + \end{array} \rightarrow (-\infty,0)\cup(\tfrac{1}{4},+\infty)$$

$$\xrightarrow{\cap} (\tfrac{1}{4},+\infty)$$

ب)
$$\left|\frac{x-2}{2x+1}\right|>1 \rightarrow -1>\frac{x-2}{2x+1}>1 \begin{cases} \frac{x-2-2x-1}{2x+1}>0 \rightarrow \frac{-x-3}{2x+1}>0 \\ \frac{x-2}{2x+1}<-1 \rightarrow \frac{x-2+2x+1}{2x+1}<0 \rightarrow \frac{3x-1}{2x+1}<0 \end{cases}$$

$$\begin{array}{c|c|c} -3 & & -\frac{1}{2} \\ - & + & - \end{array} \quad (-3,-\tfrac{1}{2})$$

$$\begin{array}{c|c|c} -\frac{1}{2} & & \frac{1}{3} \\ + & - & + \end{array} \quad (-\tfrac{1}{2},\tfrac{1}{3})$$

$$\rightarrow (-3,\tfrac{1}{3})-\{\tfrac{1}{2}\}$$

۳)
$$x^2<|x+6| \rightarrow x^2<x+6<-x^2 \begin{cases} x^2-x-6<0 \quad (x-3)(x+2)<0 \quad \begin{array}{c|c|c} -2 & & 3 \\ + & - & + \end{array} \quad (-2,3) \\ x^2+x+6<0 \quad 1-24<0 \quad \Delta<0 \text{ غ ق ق} \end{cases}$$

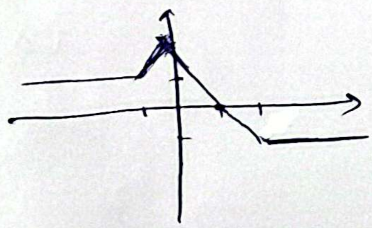
$$-2<x<3$$

$$|x-\tfrac{1}{2}|<\tfrac{5}{2}$$

$$\boxed{\alpha=\tfrac{1}{2}}$$

۴)
$$y=|x+1|-|2x|+|x-2|$$

-1		0		2
$-\cancel{x}-1+\cancel{2x}-\cancel{x}+2$	$x+1+2x-x+2$		$x+1-\cancel{2x}+\cancel{x}-2$	
1	$2x+3$	$x+1-2x-x+2$	-1	
		$-2x+3$		



$$\left.\begin{array}{l} \text{کمترین}=-1 \\ \text{بیشترین}=3 \end{array}\right\} \rightarrow 3-1=\boxed{2}$$

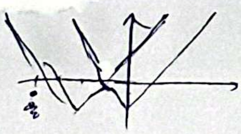
۵)
$$y=|\tfrac{1}{2}x|-2 \quad , \quad y'=|\tfrac{1}{2}x+2|-1$$

$$-\tfrac{1}{2}x-2=\tfrac{1}{2}x+\cancel{2}-1 \qquad x=-3$$

$$|\tfrac{1}{2}x|-2=|\tfrac{1}{2}x+2|-1 \qquad \begin{array}{|l} -3 \\ -\frac{1}{2} \end{array}$$

$$\tfrac{3}{2}-2,\ \tfrac{-3}{2}$$

$$\begin{array}{c|c} & -4 \\ -\frac{1}{2}x-3 & \frac{1}{2}x+1 \end{array}$$



(۶

الف) $f(x)=[|1-x|-|x+2|]$

$|1-x|-|x+2|$

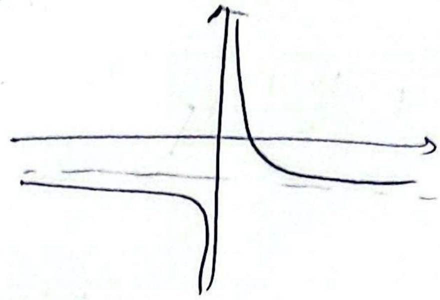
-2		$+1$	
$1-x+x+2$ 5	$1-x-x-2$ $-2x-1$		$x-1-x-2$ -5

$R_f\{-5,-4,\dots,4,5\}$

ب) $f(x)=\frac{1}{x}-[\frac{x+1}{x}]$, $\frac{1}{x}-[1+\frac{1}{x}]$, $\frac{1}{x}-[\frac{1}{x}]-1$

$f(x)=\frac{1}{x}-1$

$R_f=(-\infty,-1)\cup(-1,+\infty)$



(۷

الف) $y=[-2x]-1$ $\quad x\in(-1,1)$

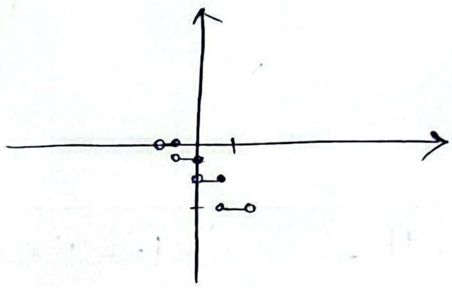
$-1<x<1 \to 2>-2x>-2 \to -2<-2x<2$

$-2<-2x<-1 \xrightarrow{[-2x]=-2} -3$ $\quad \frac{1}{2}<x<1$

$-1\le -2x<0 \xrightarrow{[-2x]=-1} -2$ $\quad 0<x\le\frac{1}{2}$

$0\le -2x<1 \xrightarrow{[-2x]=0} -1$ $\quad -\frac{1}{2}<x\le 0$

$1\le -2x<2 \xrightarrow{[-2x]=1} 0$ $\quad -1<x\le -\frac{1}{2}$



ب) $y=x-2[\frac{x}{2}]$ $\quad x\in(-2,2)$

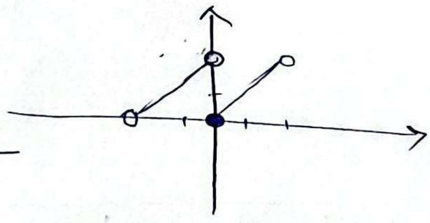
$-2<x<2 \to -1<\frac{x}{2}<1$

$-1<\frac{x}{2}<0 \xrightarrow{[\frac{x}{2}]=-1} x+2$ $\quad -2<x<0$

x	y
-2	0
0	2

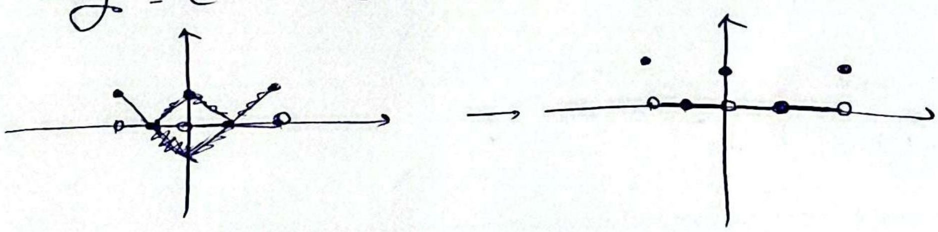
$0\le\frac{x}{2}<1 \xrightarrow{[\frac{x}{2}]=0} x$ $\quad 0\le x<2$

x	y
0	0
2	2

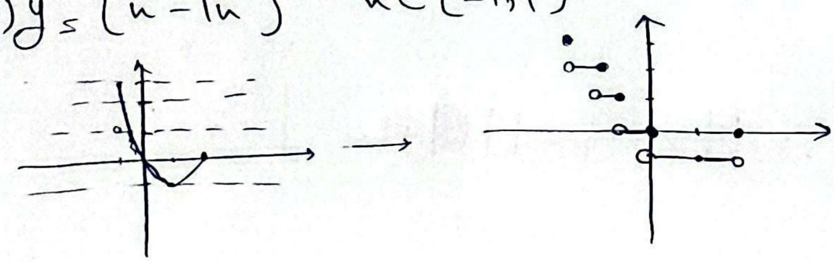


(۸

الف) $y=[||x|-1|]$ $\quad x\in[-2,2]$



ب) $y=[x^2-2x]$ $\quad x\in[-1,2]$



(۹

$a+[b]=۴,۲ \rightarrow \boxed{a=۱,۲}$ $\qquad$ $b-[a]=۲,۴ \rightarrow \boxed{b=۳,۴}$

$[a+[b]]=[۴,۲]$ $\qquad$ $[b-[a]]=[۲,۴]$

$\begin{cases} \cancel{[a]}+[b]=۴ \\ [b]\cancel{-[a]}=۲ \end{cases} \rightarrow [a]=۱$ $\qquad$ $[b]-[a]=۲$

$۲[b]=۶ \rightarrow [b]=۳$ $\qquad$ $a+b=۱,۲+۳,۴=\boxed{۴,۶}$

(۱۰

$(1+\sqrt{۲})^۶+(1-\sqrt{۲})^۶=۱۹۸$

$1-\sqrt{۲}=1-۱,۴۱=-۰,۴۱ \rightarrow -۱<1-\sqrt{۲}<۰ \rightarrow ۰<(1-\sqrt{۲})^۶<1$

$\underset{۱۹۷}{\cancel{۱۹۸-۱}}<(1+\sqrt{۲})^۶<۱۹۸ \rightarrow [(1+\sqrt{۲})^۶]=\boxed{۱۹۷}$