هستی رمضانی/ یازدهم دختر

~~۲۰~~ به دلیل تایپ کردن اسم نمره شما (۰) می باشد

①

$\frac{1}{x-1} = |x-1|$

$\to x \geq 1$: $x-1 = \frac{1}{x-1}$

$(x-1)^2 = 1 \to x^2+1-2x = 1$

$x^2-2x=0$

$x(x-2)=0$

$x=0$ غ ق ق

$x=2$ ق ق

$\to x<1$: $1-x = \frac{1}{x-1}$

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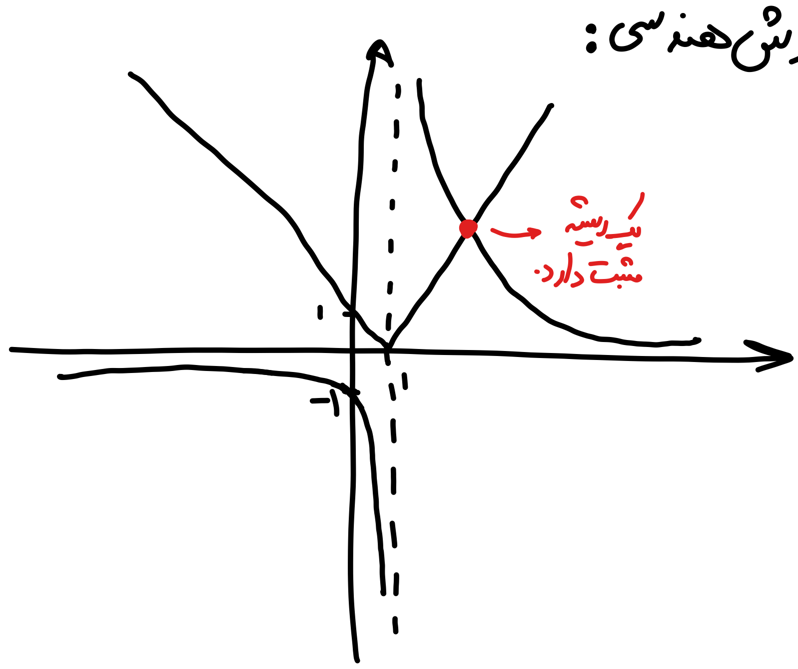
$(1-x)(x-1)=1$

$-x^2+2x-1=1 \to x^2-2x+2=0$

$\Delta<0 \to \times$ ریشه

$\Longrightarrow x=2$ یک ریشه

روش هندسی:



②

الف) $\left|\frac{2x-1}{x}\right|<2 \to -2<\frac{2x-1}{x}<2$

$-2<\frac{2x-1}{x} \to -2x<2x-1 \to 4x>1$

$\Longrightarrow x>\frac{1}{4}$

$\frac{2x-1}{x}<2 \to 2x-1<2x \to -1<0$ ✓

$\Longrightarrow x=(\frac{1}{4},+\infty)$

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ب) $\left|\frac{x-2}{2x+1}\right|>1$

$\frac{x-2}{2x+1}-1>0 \to \frac{x-2-2x-1}{2x+1}>0 \to \frac{-x-3}{2x+1}>0$ $\quad$ -3, $-\frac{1}{2}$: $-\ +\ -$ (ن) $\to x=(-3,-\frac{1}{2})$ I

$\frac{x-2}{2x+1}<-1 \to \frac{x-2+2x+1}{2x+1}<0 \to \frac{3x-1}{2x+1}<0$ $\quad$ $-\frac{1}{2}$, $\frac{1}{3}$: $+\ -\ +$ (ن) $\to x=(-\frac{1}{2},\frac{1}{3})$ II

$I \cap II \to x=(-3,\frac{1}{3})-\{-\frac{1}{2}\}$

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$x^2 < |x+6| \to x^2 < x+6 \to x^2 - x - 6 < 0 \to (x-3)(x+2) = 0 \to x=-2 ,\ x=3$

$x+6 < -x^2 \to x^2 + x + 6 < 0 \to \Delta < 0$ ✗ غق

$\Rightarrow x = (-2, 3)$

$|x-\alpha| < \beta \to |x-\alpha| - \beta < 0 \xrightarrow{x=-2} |x+2| - \beta = 0$, $\xrightarrow{x=3} |x-3| - \beta = 0 \Big\} \to |x+2| = |x-3|$ ۳

$\to x+2 = x-3 \to 2 = -3$ ✗

$\to x+2 = 3-x \to 2x = 1 \to \boxed{x = \frac{1}{2}}$

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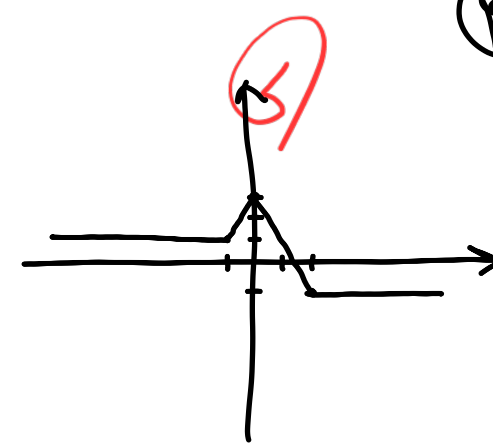
$y = |x+1| - |2x| + |x-2|$, $\quad x+1 \to -1 ,\ 2x \to 0 ,\ x-2 \to 2$

$3 - 1 = \boxed{2}$ ۲

$x<-1$	$-1<x<0$	$0<x<2$	$x>2$
$y = -x-1+2x+2-x$	$y = x+1+2x+2-x$	$y = x+1-2x+2-x$	$y = x+1-2x+x-2$
$y = 1$	$y = 2x+3$	$y = -2x+3$	$y = -1$

$x=-1$: 1 , $x=0$: 3 , $x=2$: -1



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$y = |\frac{1}{2}x| - 2 \longrightarrow y' = |\frac{1}{2}x + 2| - 1$

$|\frac{1}{2}x| - 2 = |\frac{1}{2}x + 2| - 1 \longrightarrow |\frac{1}{2}x| - |\frac{1}{2}x + 2| - 1 = 0$, $\quad \frac{1}{2}x \to 0 ,\ \frac{1}{2}x+2 \to -4$

$x<-4$	$-4<x<0$	$x>0$
$y = -\frac{1}{2}x + \frac{1}{2}x + 2 - 1$	$y = -\frac{1}{2}x - \frac{1}{2}x - 2 - 1$	$y = \frac{1}{2}x - \frac{1}{2}x - 2 - 1$
$y = 1$	$y = -x-3$	$y = -3$

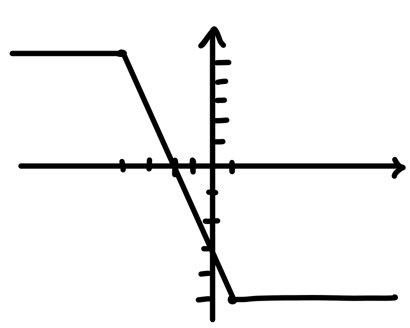
$x=-4$: 1 , $x=0$: -3

$\longrightarrow y = -x - 3 \longrightarrow -x - 3 = 0$

$\Longrightarrow \boxed{x = -3}$ ۲

۶

الف) $f(x) = [\,|1-x| - |x+4|\,]$



$\longrightarrow R_f = \{x \mid x \in [-5, 5] \wedge x \in \mathbb{Z}\}$ ۲

به عبارتی: $\{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

ب) $f(x)=\frac{1}{x}-\left[\frac{x+1}{x}\right] \to \frac{1}{x}-\left[\frac{x}{x}+\frac{1}{x}\right] \to \underbrace{\frac{1}{x}-\left[\frac{1}{x}\right]}_{R=[0,1)}-1$

$\Rightarrow R_f=[-1,0)$

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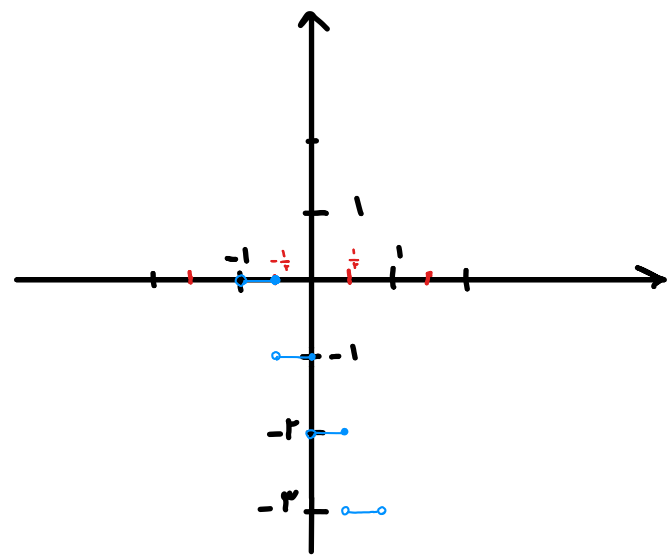
الف) $y=[-2x]-1 \qquad x\in(-1,1)$

$-1<x\le -\frac{1}{2} \to [-2x]=1 \to y=0$

$-\frac{1}{2}<x\le 0 \to [-2x]=0 \to y=-1$

$0<x\le \frac{1}{2} \to [-2x]=-1 \to y=-2$

$\frac{1}{2}<x<1 \to [-2x]=-2 \to y=-3$



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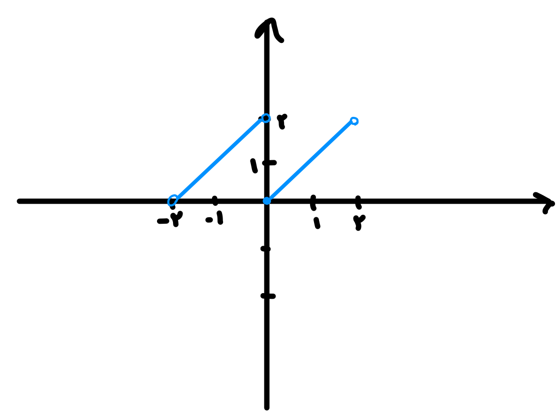
ب) $y=x-2\left[\frac{x}{2}\right] \qquad x\in(-2,2)$

$-2<x<0 \to \left[\frac{x}{2}\right]=-1 \to y=x+2$

$0<x<2 \to \left[\frac{x}{2}\right]=0 \to y=x$



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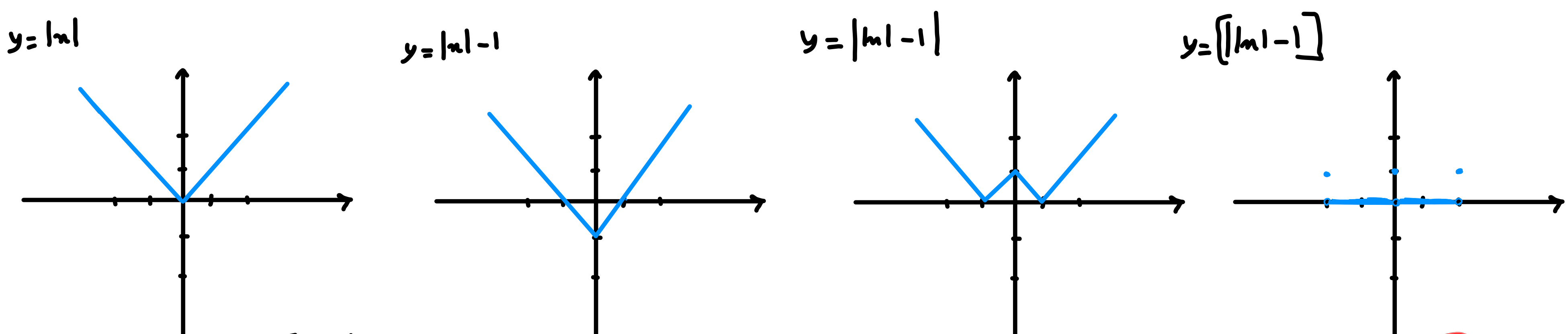
الف) $y=\left[\,\left||x|-1\right|\,\right] \qquad x\in[-2,2]$



ب) $y=[x^2-2x] \qquad x\in[-1,2]$

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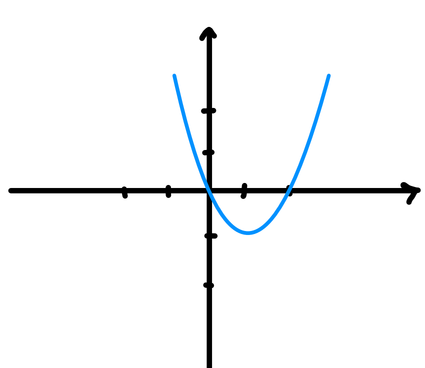
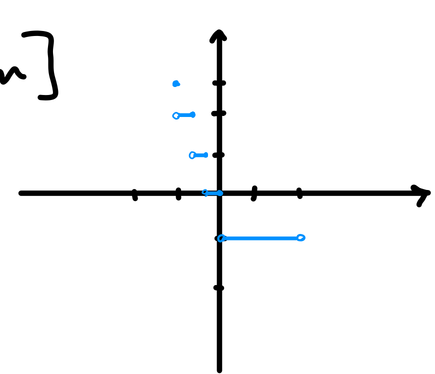
$y=x^2-2x$

رأس $\left|\begin{array}{l} -\frac{b}{2a}=1 \\ -\frac{\Delta}{4a}=-1 \end{array}\right.$



$y=[x^2-2x]$



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$a+[b]=۴,۲ \rightarrow [a+[b]]=[۴,۲] \rightarrow [a]+[b]=۴$

$b-[a]=۲,۴ \rightarrow [b-[a]]=[۲,۴] \rightarrow [b]-[a]=۲$ +

$۲[b]=۶ \rightarrow [b]=۳$

$\Rightarrow b=۳,۴$

$[a]=۱ \Rightarrow a=۱,۲$

$a+b=۱,۲+۳,۴=\boxed{۴,۶} \Longleftarrow$

۱۰

$(۱+\sqrt{۲})^۶+(۱-\sqrt{۲})^۶=۱۹۸ \rightarrow (۱+\sqrt{۲})^۶=۱۹۸-(۱-\sqrt{۲})^۶$

$-۱<۱-\sqrt{۲}<۰ \rightarrow ۰<(۱-\sqrt{۲})^۶<۱$ t

$[(۱+\sqrt{۲})^۶]=[۱۹۸-t]=[۱۹۷,\dots]=\boxed{۱۹۷}$

$۰<t<۱$