غیردهم تجربی   (14)   فاطمه ناطقی ۲۰

Date   No

1- $\frac{1}{x-1} = |x-1| \to x \neq 1 , x>1 \Rightarrow \frac{1}{(x-1)^2} = (x-1)^2 \Rightarrow 1 = (x-1)^4 \to (x-1)^4 - 1 = 0$

$\Rightarrow ((x-1)^2-1)((x-1)^2+1)=0 \Rightarrow (x-2)\times x \times (\underbrace{x^2-2x+2}_{\Delta<0})=0 \Rightarrow x=0$ — غق ; $x=2$ (2)

2- الف) $\xrightarrow{\text{توان ۲}} \frac{4x^2-4x+1}{x^2} < 4 \Rightarrow \frac{4x^2-4x+1}{x^2} - \frac{4x^2}{x^2} < 0 \Rightarrow \frac{-4x+1}{x^2} < 0 \Rightarrow$ $\begin{array}{cccc} & 0^* & \frac14 & \\ + & + & - & \end{array}$ (1,5)

$\Rightarrow x = [\frac14 , +\infty)$

ب) $\xrightarrow{\text{توان ۲}} \frac{x^2-4x+4}{4x^2+4x+1} > 1 \Rightarrow \frac{x^2-4x+4-4x^2-4x-1}{4x^2+4x+1} > 0 \Rightarrow \frac{-3x^2-8x+3}{4x^2+4x+1} \Rightarrow$ $\begin{array}{cccc} -3 & -\frac12^* & \frac13 & \\ + & - & - & + \end{array}$

$-2 < \frac{2x-1}{x} < 2 \Rightarrow \begin{cases} \frac{-1}{x} > -4 \to x > \frac14 \\ \frac{-1}{x} < 0 \to x > 0 \end{cases} \to x > \frac14$ $-3\frac{(x+3)(3x-1)}{(2x+1)^2}$ $-\frac12$ $\Rightarrow x = (-\infty, -3) \cup (\frac13, ...$

3- $x^2 < |x+6| \to |x-\alpha| < \beta$

توان ۲: $x^4 < x^2 + 12x + 36 \to x^4 - x^2 - 12x - 36 < 0 \to (\underbrace{x^2+x+6}_{\Delta<0})(\underbrace{x^2-x-6}_{(x-3)(x+2)}) < 0$

$\to \begin{array}{ccc} -2 & & 3 \\ + & - & + \end{array} \Rightarrow x \to (-2, 3) \Rightarrow b = 3 - \frac12 = \frac52$ (2)

$\alpha = \frac{3-2}{2} = \frac12 \Rightarrow |x - \frac12| < \frac52 \Rightarrow \alpha = \frac12$

4- $x = -1, 0, 2 \xrightarrow{\text{صفرهای قدرمطلق}}$

$x \le -1 \to y = -x - 1 - 2x - x + 2 \Rightarrow y = -4x + 1$

$-1 \le x \le 0 \to y = x + 1 + 2x - x + 2 = 2x + 3$

$0 \le x \le 2 \to y = 3 - 2x$

$2 \le x \to y = -1$

$\Rightarrow \left. \begin{array}{l} \max = 3 \\ \min = -1 \end{array} \right\} \Rightarrow 3 - 1 = 2$ (2)

5- $y = \frac12|x| - 2 \to y = \frac12|x+4| - 2 \to y = \frac12|x+4| - 1$

$\to \frac12|x+4| - 1 = \frac12|x| - 2 \Rightarrow -\frac12|x+4| + \frac12|x| = +1 \to |x+4| + |x| = 2 \Rightarrow \boxed{x = -3}$

(2)

۶- الف) $f(x) = [\,|1-x| - |x+4|\,] \to x = 1, -4$

$x < -4 \to [1 - x + x + 4] = [5] = 5$

$-4 \le x < 1 \to [1 - x - x - 4] = [-2x - 3] \to A = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$ $\Rightarrow R_f = A$

$1 \le x \to [x - 1 - x - 4] = [-5] = -5$

1,5

ب) $f(x) = \frac{1}{x} - \left[\frac{x+1}{x}\right]$

$f(x) = \frac{1}{x} - \left[1 + \frac{1}{x}\right] = \frac{1}{x} - \left[\frac{1}{x}\right] - 1 \Rightarrow$

$\Rightarrow R_f = [0, 1) - 1 \to [-1, 0)$

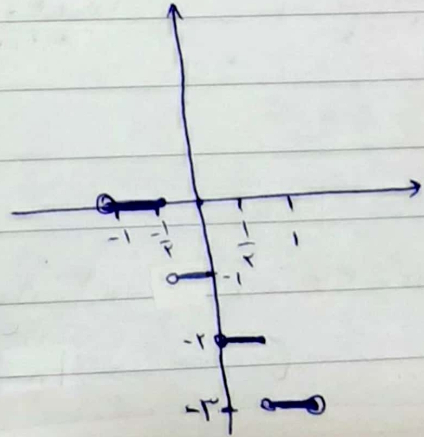
۷- الف) $y = [-2x] - 1 \quad x \in (-1, 1)$

$-1 < x \le -\frac{1}{2} \to [-2x] = 1 \Rightarrow y = 0$

$-\frac{1}{2} < x \le 0 \to [-2x] = 0 \Rightarrow y = -1$

$0 < x \le \frac{1}{2} \to [-2x] = -1 \Rightarrow y = -2$

$\frac{1}{2} < x < 1 \to [-2x] = -2 \Rightarrow y = -3$



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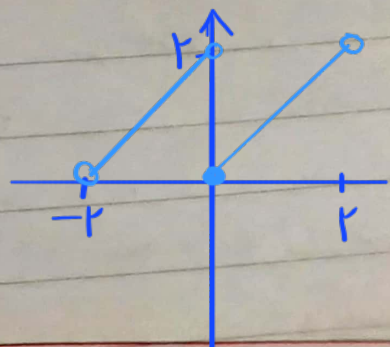
۸- ب) $f(x) = x - 2\left[\frac{x}{2}\right] \quad x \in (-2, 2)$

$0 \le x < 2 \to x$

$-2 < x < 0 \to x + 2$

$-2 < x < -1 \to \left[\frac{x}{2}\right] =$



۹- $a+[b]=۴/۲ \Rightarrow a-[a]=۰/۲$

$b-[a]=۲/۴ \Rightarrow b-[b]=۰/۴$

$\Rightarrow [b]+[a]=۴ \Rightarrow a+b=۴/۶$

$۰/۴+۰/۲=۰/۶<۱$

۳

۱۰- $\left((۱-\sqrt{۲})^۲+(۱+\sqrt{۲})^۲\right)\left((۱+\sqrt{۲})^۴+(۱-\sqrt{۲})^۴-(۱-\sqrt{۲})^۲(۱+\sqrt{۲})^۲\right)$

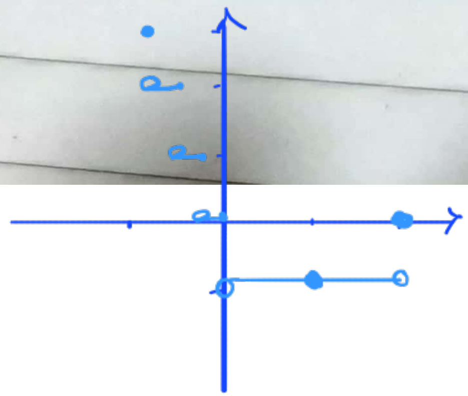
$=۶\times(۱۷+۶\sqrt{۸}+۱۷-۶\sqrt{۸}-۱)=۶\times۳۳=\boxed{۱۹۸}$

$۰<(۱-\sqrt{۲})^۴<۱$

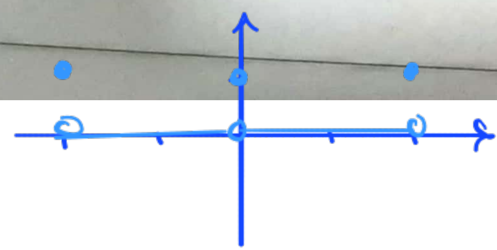
$[(۱+\sqrt{۲})^۴]=[۱۹۸-(۱-\sqrt{۲})^۴]$

$=[(۱+\sqrt{۲})^۴]=۱۹۸+[-\underbrace{(۱-\sqrt{۲})^۴}_{۰^-}]=۱۹۸-۱=۱۹۷$

۸ ب)



۸ الف)



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