

تازه طهران - تلف ۲۰ - یازدهم دهر

$$\frac{1}{x-1} = |x-1| \xrightarrow{x > 1} \frac{1}{x-1} = x-1 \Rightarrow (x-1)^2 = 1$$

$$\Rightarrow x^2 - 2x + 1 = 1 \Rightarrow x^2 - 2x = 0 \Rightarrow x = 0$$

غیر

$$\xrightarrow{x < 1} \frac{1}{x-1} = 1-x \Rightarrow$$

$$-x^2 + 2x - 1 = 1 \Rightarrow x^2 - 2x + 2 = 0$$

$$\Delta = b^2 - 4ac = 4 - 8 = -4$$

$$\Rightarrow x = 2 : \text{مطلوبه}$$

$$\text{الف) } \left| \frac{2x-1}{x} \right| < 2 \Rightarrow -2 < \frac{2x-1}{x} < 2 \Rightarrow$$

$$-2 < \frac{2x-1}{x} + 2 \Rightarrow \frac{4x-1}{x} > 0 \quad \text{①}$$

$$\Rightarrow \frac{2x-1}{x} - 2 < 0 \Rightarrow \frac{-1}{x} < 0 \Rightarrow x > 0 \quad \text{②}$$

$$\text{①} \cap \text{②} \rightarrow x > \frac{1}{4} \quad \text{ب) } \left| \frac{x-2}{2x+1} \right| > 1 \Rightarrow$$

$$\frac{x-2}{2x+1} > 1 \Rightarrow \frac{x-2}{2x+1} - 1 > 0$$

$$\Rightarrow \frac{x-2-2x-1}{2x+1} > 0 \Rightarrow \frac{-x-3}{2x+1} > 0 \quad \text{③}$$

$$\frac{x-2}{2x+1} < -1 \Rightarrow \frac{x-2}{2x+1} + 1 < 0 \Rightarrow \frac{x-2+2x+1}{2x+1} < 0$$

$$\Rightarrow \frac{3x-1}{2x+1} < 0 \quad \text{④}$$

$$x \in (-3, -\frac{1}{2}) \cup (-\frac{1}{2}, \frac{1}{3})$$

$$x^r < |x+4| \Rightarrow x+4 > x^r \Rightarrow x^r - x - 4 < 0 \quad -2$$

$$\Rightarrow (x-3)(x+2) < 0 \quad \begin{array}{c} -2 \quad 3 \\ + \quad - \quad + \end{array} \quad ①$$



$$x+4 < -x^r \Rightarrow x^r + x + 4 < 0$$

$$\Delta = 1 - 16 < 0 \quad \text{⤵}$$

$$|x-a| < \beta \Rightarrow -\beta < x-a < \beta \xrightarrow{+a} x-\beta < x < x+\beta \quad ②$$

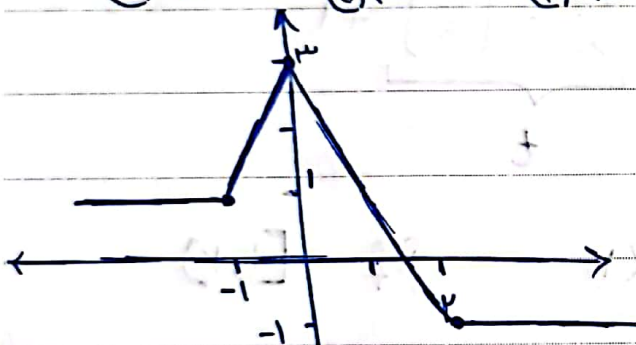
$$①, ② \Rightarrow \begin{cases} x-\beta = -2 \\ x+\beta = 3 \end{cases} \Rightarrow 2x = 1 \Rightarrow x = \boxed{\frac{1}{2}}$$

$$y = |x+1| - |2x| + |x-2| \quad \begin{array}{c} -1 \quad 0 \quad 2 \\ \hline -x-1+2x \quad x+1+2x \quad x+1-2x \quad x+1-2x+x-2 \\ -x+2 \quad -x+2 \quad -x+2 \quad = -1 \\ = 1 \quad = 2x+3 \quad = -2x+3 \end{array}$$

① ⤵

② ⤵

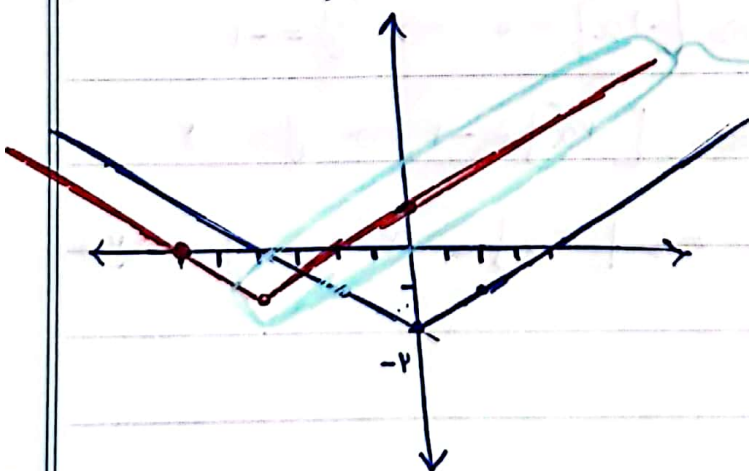
③ ⤵



$$\text{کثرین} + \text{بسترن} = -1 + 3 = \boxed{2}$$

$$y = \left| \frac{1}{2}x \right| - 2 \quad \begin{array}{c} 0 \\ \hline -\frac{1}{2}x - 2 \quad \frac{1}{2}x - 2 \end{array} \quad -2$$

⤵

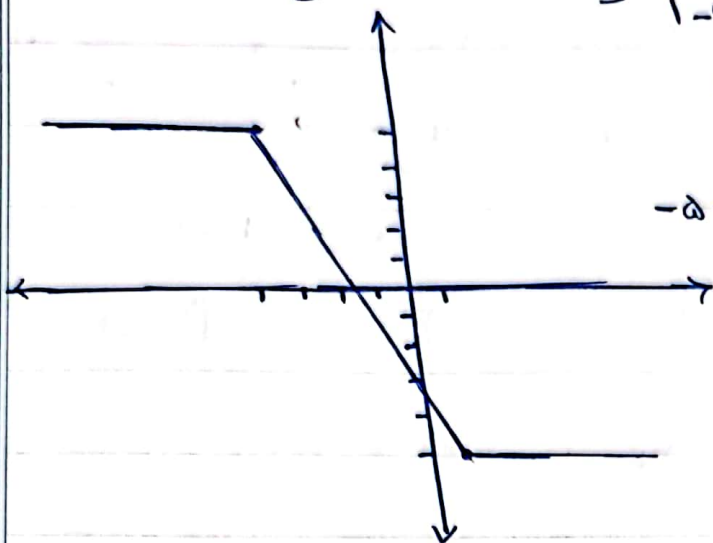


$$a = \frac{1}{2} \quad y = \frac{1}{2}x + 1 \quad \text{⤵}$$

$$\frac{1}{2}x + 1 = -\frac{1}{2}x - 2$$

$$\boxed{x = -3}$$

الف) $f(x) = [1-x] - [x+\varepsilon]$



$$\begin{array}{ccc} -\varepsilon & 1 & -\varepsilon \\ \hline 1-x+x+\varepsilon & 1-x-x-\varepsilon & x-1-x-\varepsilon \\ = 0 & = -2x-1 & = -0 \end{array}$$

$$-\omega \leq |1-x| - |x+\varepsilon| \leq \omega$$

$$\Rightarrow [1-x] - [x+\varepsilon] =$$

$$\{-\omega, -\varepsilon, \dots, \varepsilon, \omega\}$$

$$\Rightarrow R_f = \{\pm\omega, \pm\varepsilon, \pm\omega, \pm\varepsilon, \pm 1, 0\}$$

ب) $f(x) = \frac{1}{x} - \left[\frac{x+1}{x} \right] = \frac{1}{x} - \left[\frac{1}{x} \right] - 1$

$$\Rightarrow 0 \leq t < 1 \Rightarrow -1 \leq t-1 < 0 \Rightarrow R_f = [-1, 0]$$

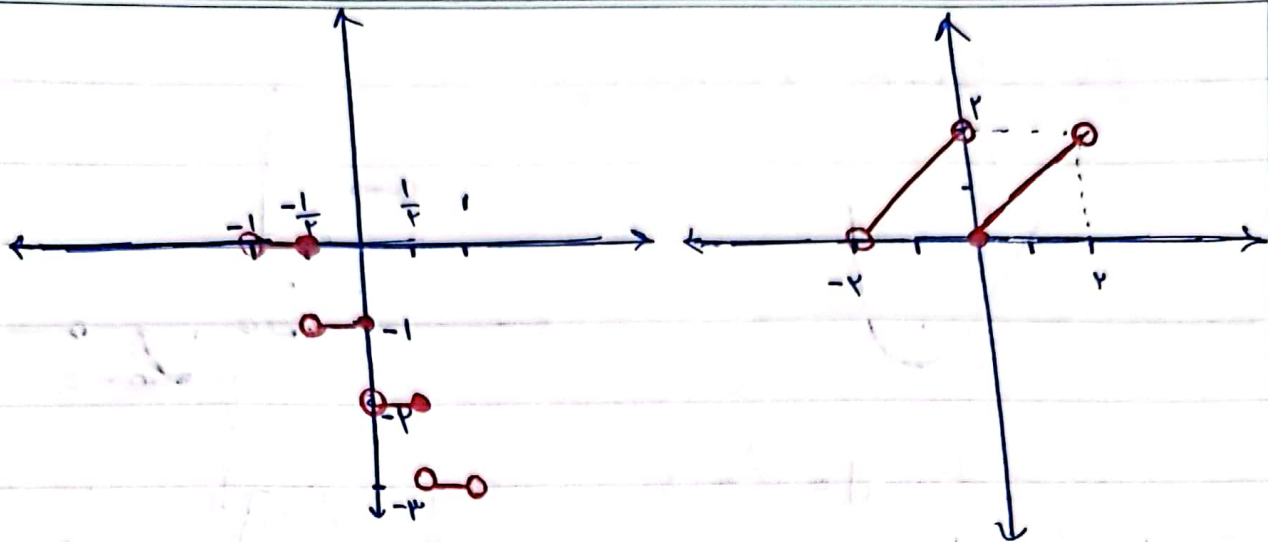
الف) $y = [-2x] - 1$

$$-1 < x \leq -\frac{1}{2} \Rightarrow 1 \leq -2x < 2 \Rightarrow [-2x] = 1 \Rightarrow y = 0$$

$$-\frac{1}{2} < x \leq 0 \Rightarrow 0 \leq -2x < 1 \Rightarrow [-2x] = 0 \Rightarrow y = -1$$

$$0 < x \leq \frac{1}{2} \Rightarrow -1 \leq -2x < 0 \Rightarrow [-2x] = -1 \Rightarrow y = -2$$

$$\frac{1}{2} < x < 1 \Rightarrow -2 < -2x < -1 \Rightarrow [-2x] = -2 \Rightarrow y = -3$$

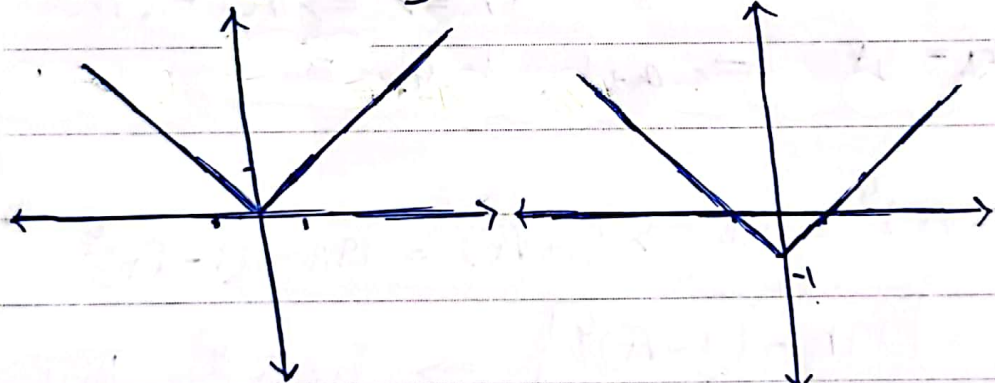


ب) $y = x - 2 \left[\frac{x}{2} \right]$

$$-2 < x < 0 \Rightarrow -1 < \frac{x}{2} < 0 \Rightarrow \left[\frac{x}{2} \right] = -1 \Rightarrow y = x + 2$$

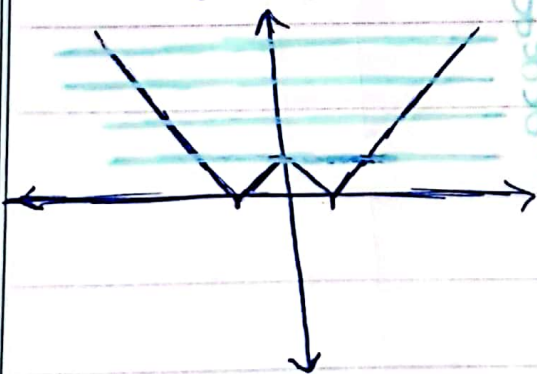
$$0 \leq x < 2 \Rightarrow 0 \leq \frac{x}{2} < 1 \Rightarrow \left[\frac{x}{2} \right] = 0 \Rightarrow y = x$$

الف) $y = [|x| - 1]$ $x \in [-2, 2]$

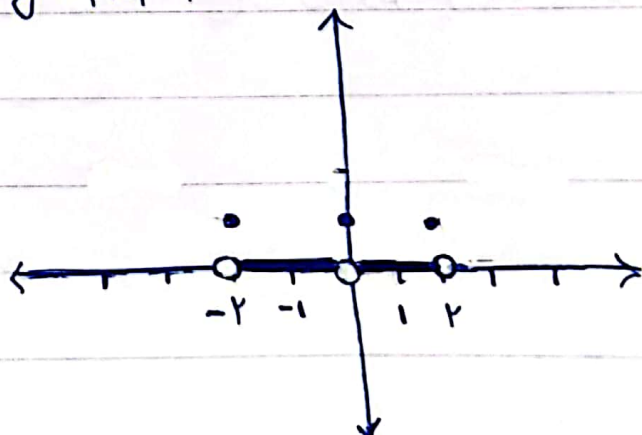


$$y = |x|$$

$$y = |x| - 1$$

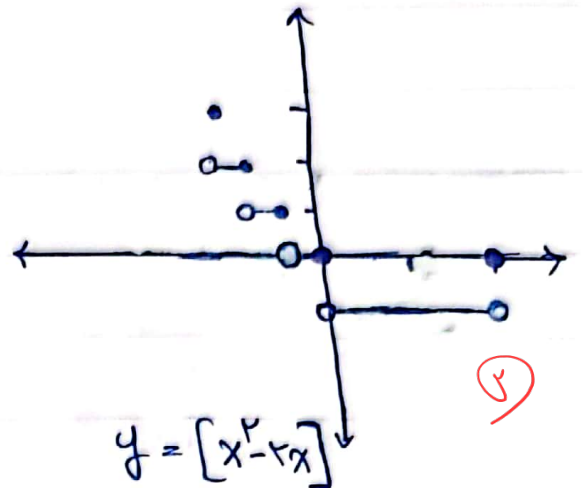
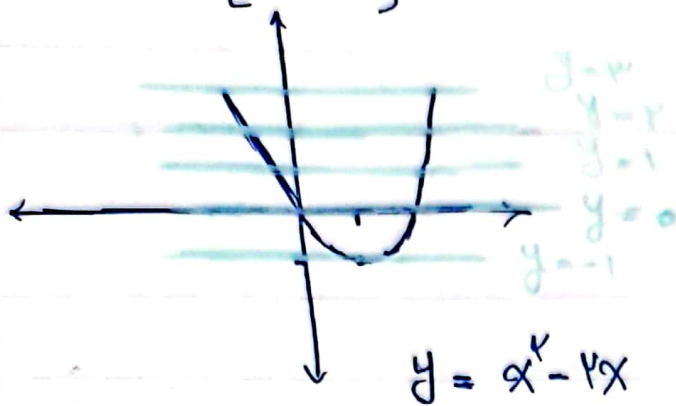


$$y = ||x| - 1|$$



Subo $y = [|x| - 1]$

ب) $y = [x^2 - 2x] \quad x \in [-1, 2]$



$x_s = 1 \quad y_s = -1$

$a + [b] = \varepsilon, 2$

$[a] = m \Rightarrow a = m + 0,2$

$b - [a] = 2, \varepsilon$

$[b] = n \Rightarrow b = n + 0, \varepsilon$

$\Rightarrow m + 0,2 + n = \varepsilon, 2 \Rightarrow m + n = \varepsilon$

$\Rightarrow n + 0, \varepsilon - m = 2, \varepsilon \Rightarrow n - m = 2$

$2n = 4 \Rightarrow n = 2, m = 1$

$b = 2, \varepsilon, a = 1,2 \Rightarrow a + b = \boxed{\varepsilon, 4}$

$(1 + \sqrt{2})^4 + (1 - \sqrt{2})^4 = 198 \Rightarrow (1 + \sqrt{2})^4 = 198 - (1 - \sqrt{2})^4$

$[(1 + \sqrt{2})^4] = [198 - (1 - \sqrt{2})^4] \Rightarrow [(1 + \sqrt{2})^4] = \boxed{197}$

$t < 1$ ← به توان زوج می رسد و در صورت زوج می شود