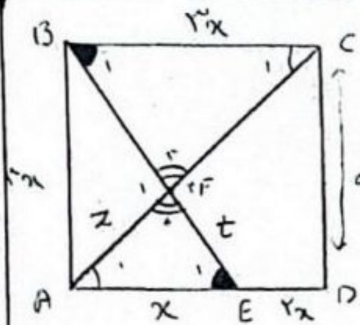
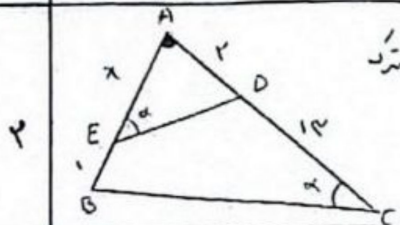
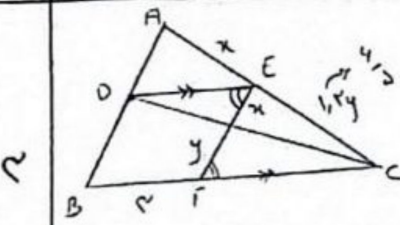




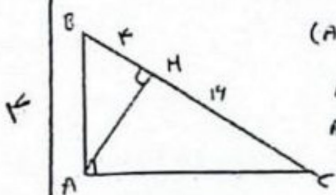
$\hat{F}_1, \hat{F}_2, \hat{F}_3, \hat{F}_4$ متساوی: $\hat{B}_1, \hat{E}_1$ $\rightarrow \triangle BFC \sim \triangle AFE \rightarrow \frac{BC}{AE} = \frac{BF}{FE} = \frac{CF}{FA}$
 $\Rightarrow CF = \frac{AF}{2}, BF = \frac{FE}{2}$
 $AC = CF + AF = \frac{r}{2} + r = \frac{3r}{2} \rightarrow (AC)^2 = 9x^2 + 9x^2 + 14z^2 \rightarrow z = \frac{\sqrt{14}}{6}x$
 $BE = BF + EF = \frac{r}{2} + t = t \rightarrow (BE)^2 = 9x^2 + x^2 + 14t^2 \rightarrow t = \frac{\sqrt{14}}{6}x$
 $\Rightarrow \frac{EF}{AF} = \frac{t}{r} = \frac{\frac{\sqrt{14}}{6}x}{\frac{r}{2}} = \frac{\sqrt{14}}{3}$



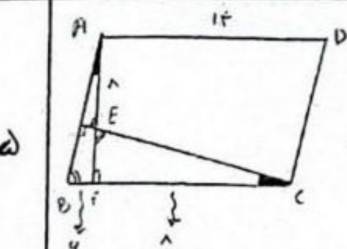
$\hat{A} = \hat{A} \rightarrow \triangle AED \sim \triangle ABC \rightarrow \frac{AE}{AC} = \frac{AD}{AB} \rightarrow \frac{x}{10} = \frac{y}{20}$
 $\hat{AED} = \hat{ACB}$
 $\rightarrow x^2 + z^2 = y^2 \rightarrow x^2 + z^2 = 100 \rightarrow (x+4)(x-4) = 0 \rightarrow \begin{cases} x=4 \\ x=10 \end{cases}$



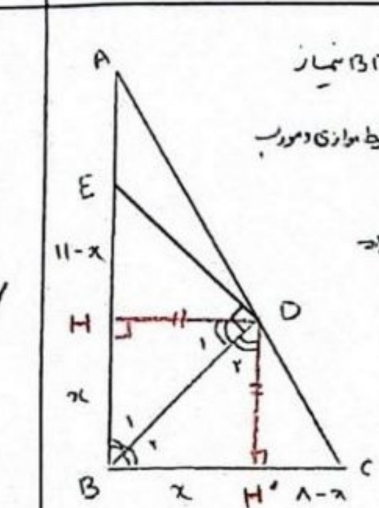
$xy = 2x \rightarrow x = \frac{y}{2}$
 $DE \parallel FC \rightarrow \frac{DE}{FC} = \frac{x}{y} \rightarrow \frac{DE}{FC} = \frac{\frac{y}{2}}{y} \rightarrow DE = \frac{r}{2} FC$
 $DE \parallel BC \rightarrow \frac{DE}{BC} = \frac{AE}{AC} \rightarrow \frac{\frac{r}{2} FC}{r+FC} = \frac{x}{x+y} \rightarrow \frac{\frac{r}{2} FC}{r+FC} = \frac{\frac{r}{2} y}{\frac{r}{2} y + y} \rightarrow FC = \frac{10}{r}$
 $\rightarrow BC = r + \frac{10}{r} = \frac{r^2 + 10}{r} = \frac{16}{r} = 4, \sqrt{10}$



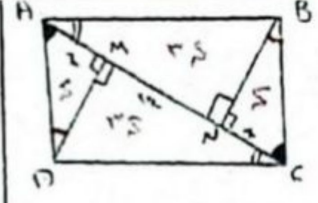
$(AH)^2 = x \cdot y \rightarrow AH = \sqrt{x}$
 $\hat{AHC} = \sqrt{14^2 + 1^2} = \sqrt{197} \rightarrow AC$
 $\hat{ABH} = \sqrt{4^2 + 1^2} = \sqrt{17} \rightarrow AB$
 $\xrightarrow{\text{نسبت}} 2$



$\triangle ABF \sim \triangle CEF \rightarrow \frac{EF}{BF} = \frac{FC}{AF} \rightarrow EF \cdot (x + EF) = x^2$
 $\Rightarrow (EF)^2 + xEF - x^2 = 0 \rightarrow (EF + 11)(EF - 1) = 0 \Rightarrow \begin{cases} EF = 11 \\ EF = 1 \end{cases}$
 $AF = x + 11$



$\triangle BDH \sim \triangle DH' \rightarrow BH = BH' = x$
 $\hat{D}_1, \hat{B}_1, \hat{D}_2, \hat{B}_2 \rightarrow \hat{D}_1, \hat{B}_1, \hat{D}_2, \hat{B}_2 = 45^\circ \rightarrow \triangle HDH'$
 $\hat{D}_1 + \hat{B}_2 = 90^\circ, \hat{D}_2 + \hat{B}_1 = 90^\circ$
 $\Rightarrow DH = DH', BH = BH' = x$
 $\triangle ADE: (DH)^2 = BH \cdot HE \rightarrow x^2 = x(11-x) \rightarrow x = 11$
 $\triangle ABC: HD \parallel BC \xrightarrow{\text{تقسیم}} \frac{AH}{AB} = \frac{HD}{BC}$
 $\rightarrow \frac{AE + \frac{11}{2}}{AE + 11} = \frac{\frac{11}{2}}{11} \rightarrow \triangle ADE \sim \triangle ABC \rightarrow AE = \frac{55}{2}$

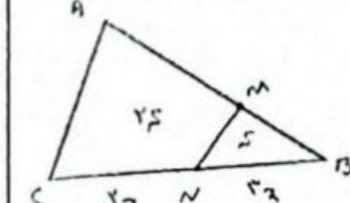


$\frac{S_{MDC}}{S_{ADM}} = \frac{r}{x} \Rightarrow r = \frac{S_{MDC}}{S_{ADM}} \cdot x$

$\left. \begin{array}{l} \hat{M}AN = \hat{B}CN \\ \hat{B}NC = \hat{D}MA \\ MN = NC \end{array} \right\} \Rightarrow \triangle MND \cong \triangle BNC$

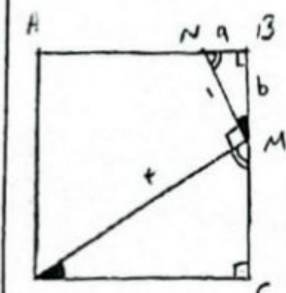
$\left. \begin{array}{l} \hat{B}AN = \hat{M}CD \\ \hat{C}MD = \hat{M}AB \\ AN = CM \end{array} \right\} \Rightarrow \triangle ANM \cong \triangle MDC$

$\Rightarrow \frac{S_{ABCD}}{S_{NBC}} = \frac{(r+x+1+x)S}{S} = 1$



$\frac{S_{MNB}}{S_{ABC}} = \frac{\frac{1}{2} \times \sin \hat{B} \times BM \times BN}{\frac{1}{2} \times \sin \hat{B} \times BA \times BC} \Rightarrow \frac{1}{4} = \frac{r \cdot BM}{\Delta BA} \Rightarrow \frac{BM}{BA} = \frac{\Delta}{9}$

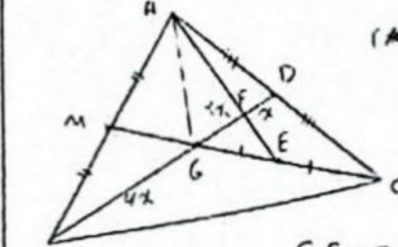
$\xrightarrow{\text{جاءنا}} \frac{BM}{AM} = \frac{\Delta}{r}$



$\frac{MD}{MN} = \frac{DC}{BM} = \frac{MC}{NB} \rightarrow \frac{r}{1} = \frac{DC}{b} \Rightarrow DC = rb \Rightarrow BC = b + MC = rb$

$(rb)^2 + (rb)^2 = 14^2 + 9^2 + 14 \Rightarrow b = \frac{r}{2}$

$\text{منه } DC = r > \frac{r}{2} > \frac{14}{2} \rightarrow S > \left(\frac{14}{2}\right)^2 = 101 \frac{1}{4}$



$(*) \triangle AGC : \frac{AF}{FE} = \frac{GF}{FD} \Rightarrow GF = rFD$

$\triangle ABC : \frac{BD}{GD} = r \rightarrow \frac{BD}{rFD} = r \rightarrow \frac{BD}{FD} = r^2$

$\left. \begin{array}{l} GE = EC \Rightarrow AE \\ \text{منه } G = CM, BD \Rightarrow AD, DC \Rightarrow GD \\ \text{منه } \end{array} \right\} \Rightarrow \text{منه } F (*)$

$\triangle AGC$