نام و نام خانوادگی ......[illegible]...... پاسخنامه تشریحی تکلیف شماره ...۲۱... کلاس ...[illegible]...

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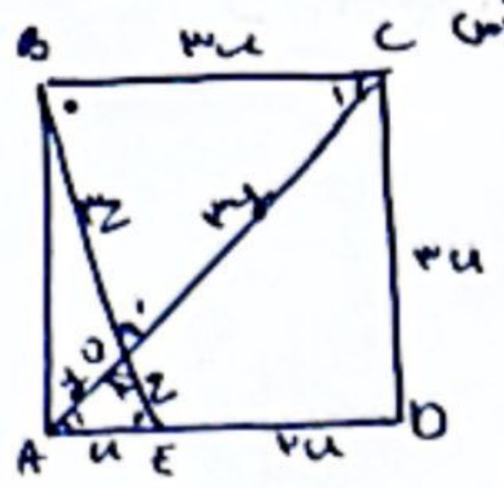


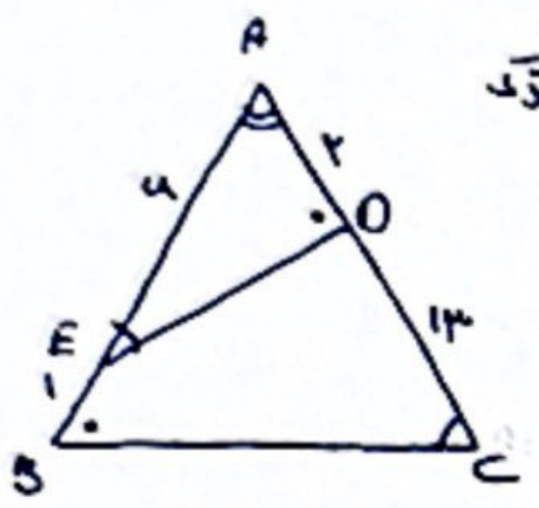
متقابل به رأس $\hat{O}_1 = \hat{O}_2$ ، مورب AC $\hat{A}_1 = \hat{C}_1$ $\xrightarrow[\text{(ز ز)}]{\text{تشابه دو زاویه}}$ $A\hat{O}E \sim B\hat{O}C \Rightarrow \frac{BC}{AE} = \frac{OC}{OA} = \frac{BO}{OE}$ → $OC = 3OA$ ، $BO = 3OE$

– $A\hat{C}D$ فیثاغورث در: $AC^2 = CD^2 + AD^2 \Rightarrow 16y^2 = (3x)^2 + (3x)^2 \Rightarrow 16y^2 = \underbrace{2\times 9x^2}_{18x^2}$ $\xrightarrow{\text{جذر}}$ $4y = 3\sqrt{2}x \Rightarrow y = \frac{3\sqrt{2}x}{4}$

– $A\hat{B}E$ فیثاغورث در: $BE^2 = AB^2 + AE^2 \Rightarrow 16z^2 = (x)^2 + (3x)^2 \Rightarrow 16z^2 = \underbrace{x^2 + 9x^2}_{10x^2}$ $\xrightarrow{\text{جذر}}$ $4z = \sqrt{10}x \Rightarrow z = \frac{\sqrt{10}x}{4}$

$\frac{OE}{OA} = \frac{z}{y} = \frac{\sqrt{10}\cancel{x}}{\cancel{4}} \times \frac{\cancel{4}}{3\sqrt{2}\cancel{x}} = \frac{1}{3} \times \sqrt{\frac{10}{2}} = \frac{\sqrt{5}}{3}$ ← جواب

۲

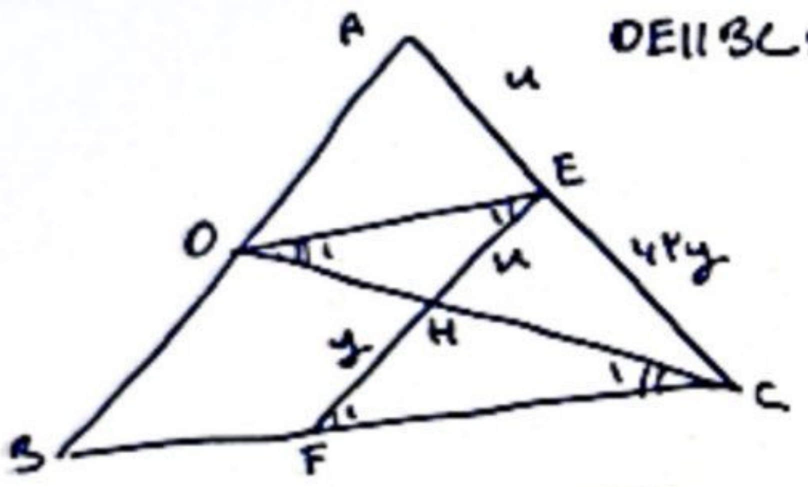


$A\hat{E}D = A\hat{C}B$ ، مشترک $\hat{A} = \hat{A}$ $\xrightarrow[\text{(ز ز)}]{\text{تشابه دو زاویه}}$ $A\hat{E}D \sim A\hat{B}C \Rightarrow \frac{AE}{AC} = \frac{ED}{BC} = \frac{AD}{AB} \Rightarrow \frac{x}{15} = \frac{2}{x+1} \Rightarrow$

$x^2 + x = 30 \Rightarrow x^2 + x - 30 = 0 \Rightarrow (x+6)(x-5) = 0$ → $\boxed{x=5}$ ، $x=-6$ غ ق ق

جواب ↑

۳



$DE \parallel BC$ ، $3y = 5x$ ، $BF = 3$ ، $BC = ?$

↳ $0.6y = x$ ⊛

$\frac{x}{y}$ ⊛ $\frac{0.6y}{y} = \frac{6}{10}$

$DE \parallel BC \Rightarrow \hat{E}_1 = \hat{F}_1$ ، EF / CD / $\hat{C}_1 = \hat{D}_1$ $\xrightarrow{\text{(ز ز)}}$ $H\hat{D}E \sim H\hat{F}C \Rightarrow \frac{DE}{FC} = \frac{HD}{HC} = \frac{EH}{HF}$ → $OE = 0.6FC$ ، $HD = 0.6HC$

$DE \parallel BC \Rightarrow A\hat{D}E \sim A\hat{B}C \Rightarrow \frac{DE}{BC} = \frac{AE}{AC} = \frac{AD}{AB}$ → $BC = 3ED$ ، $AB = 3AD$

$\frac{1}{3} = \frac{0.6\cancel{y}}{1.8\cancel{y}}$ ⊛ $\frac{x}{x+1.2y}$

$BC = 3DE$ ، $DE = 0.6FC$ $\Rightarrow BC = 3DE \Rightarrow FC + BF = 3(0.6FC) \Rightarrow FC + 3 = 1.8FC$

$\Rightarrow 1.8FC - FC = 3 \Rightarrow FC = \frac{30}{8} \Rightarrow FC = 3.75$ I

$BF = 3$ II

$I + II = 3.75 + 3 = 6.75$ ← جواب

↳ $BC = BF + FC$

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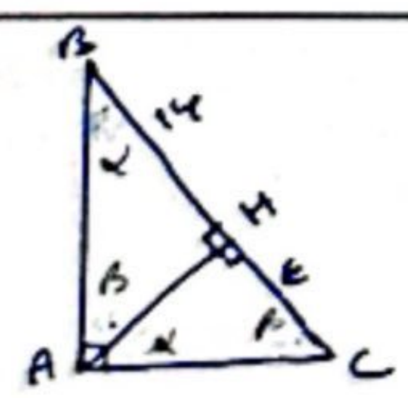

$BH = 20 - 4 = 16$

$HC = 4$ ← کمترین فاصله H از رأس مجاور

$AH^2 = HC \times BH \Rightarrow AH^2 = \underbrace{4 \times 16}_{64} \Rightarrow AH = 8$

$B\hat{A}H \sim A\hat{H}C \Rightarrow \frac{AB}{AC} = \frac{BH}{AH} \Rightarrow \frac{AB}{AC} = \frac{16}{8} = 2$ ← جواب

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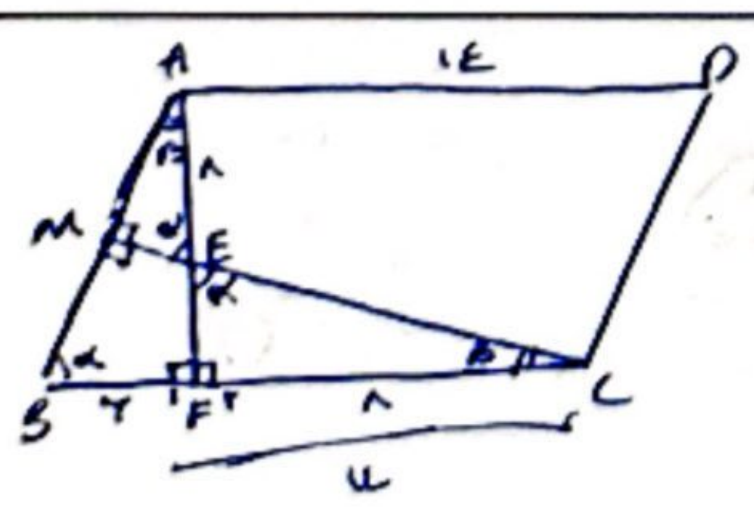

$B\hat{A}F \sim E\hat{C}F \Rightarrow \frac{BF}{EF} = \frac{AB}{EC} = \frac{AF}{CF} \Rightarrow \frac{4}{x} = \frac{x+8}{8} \Rightarrow x^2 + 8x = 48$

$\Rightarrow x^2 + 8x - 48 = 0 \Rightarrow (x-4)(x+12)$ ، $x = -12$ غ ق ق

$\boxed{x=4}$

$\Rightarrow AF = x + 8 \xrightarrow{x=4} 4 + 8 = 12$ ← جواب

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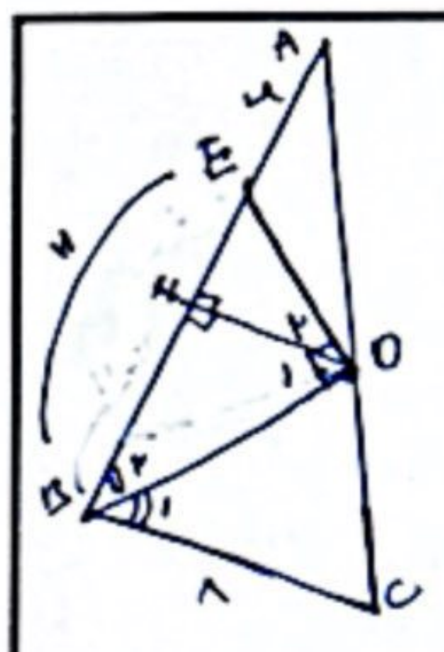

$\hat{B}_1 = \hat{B}_2$

$HO \parallel BC \Rightarrow \hat{O}_1 = \hat{B}_1 \Rightarrow \hat{B}_2 = \hat{O}_1 \Rightarrow HO = BH$ (*) , $\hat{H} = 90 \to \frac{180-90}{2} = 45^\circ \to \hat{D}_1 = 45^\circ$, $\hat{B}_2 = 45^\circ$

$\hat{B}_1 = \hat{B}_2$ چون نیمساز ، $\hat{B}_2 = \hat{B}_1 = 45^\circ$

مثلث قائم‌الزاویه BOE : $EH \times BH = DH^2 \xrightarrow{(*)} EH \times \cancel{BH} = \frac{BH^{\cancel{2}}}{BH} \Rightarrow EH = BH = \frac{11}{2} = HD$ ، $HD = BH$ (*)

$HO \parallel BC \xRightarrow{\text{تالس}} \frac{AH}{AB} = \frac{HD}{BC} \Rightarrow \frac{x+\frac{11}{2}}{x+11} = \frac{\frac{11}{2}}{8}$

$\Rightarrow \frac{x+\frac{11}{2}}{x+11} = \frac{11}{16} \Rightarrow 11x + 121 = 16x + 88 \Rightarrow x = 6.6$ ← جواب

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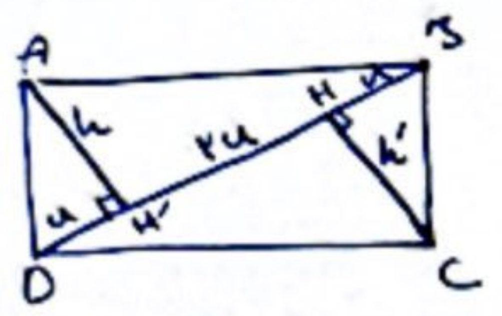

$A\hat{O}B$ در h $\Rightarrow h^2 = OH' \times H'B \Rightarrow h^2 = x \times 3x \Rightarrow h^2 = 3x^2 \Rightarrow h = \sqrt{3}\,x$

$B\hat{C}D$ در h' $\Rightarrow h'^2 = BH' \times HD \Rightarrow h'^2 = 3x \times x \Rightarrow h'^2 = 3x^2 \Rightarrow h' = \sqrt{3}\,x$ $\Big\} \Rightarrow h = h'$

S مستطیل $= S_{A\hat{B}D} + S_{B\hat{D}C} \xRightarrow{*} 2 \times S_{A\hat{B}D} = \cancel{2} \times \frac{\sqrt{3}x \times 4x}{\cancel{2}} = 4\sqrt{3}x^2$ *

← چون ارتفاع مثلث ABD و DBC با هم برابر و قاعده آن ها مشترک یعنی: $S_{A\hat{B}D} = S_{B\hat{D}C}$ *

مساحت مثلث ها مساوی: $S_{A\hat{B}D} = S_{B\hat{D}C} \Rightarrow \frac{h \times x}{2} = \frac{\sqrt{3}x^2}{2}$ * ، $h = \sqrt{3}x$

لازم و $h' = h$

$x = HB = DH'$ قاعده ها برابر

$\frac{*}{*} = 4\sqrt{3}\cancel{x^2} \times \frac{2}{\sqrt{3}\cancel{x^2}} = 8 \to$ جواب

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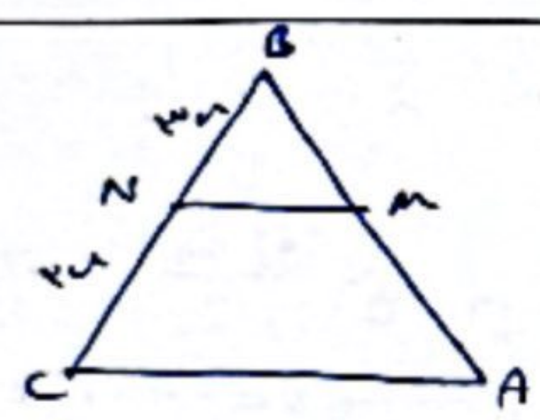

$\frac{S_{A\hat{B}C}}{S_{B\hat{M}N}} = 3 \Rightarrow \frac{BC \times AB \times \cancel{\sin\hat{B}}}{BN \times MB \times \cancel{\sin\hat{B}}} = 3 \Rightarrow \frac{5\cancel{x} \times AB}{3\cancel{x} \times MB} = 3 \Rightarrow \frac{AB}{MB} = \frac{9}{5}$

$\Rightarrow \frac{BM}{AB} = \frac{5}{9} \xrightarrow{\text{تفضیل در صورت}} \frac{BM}{AM} = \frac{5}{4}$ ← جواب

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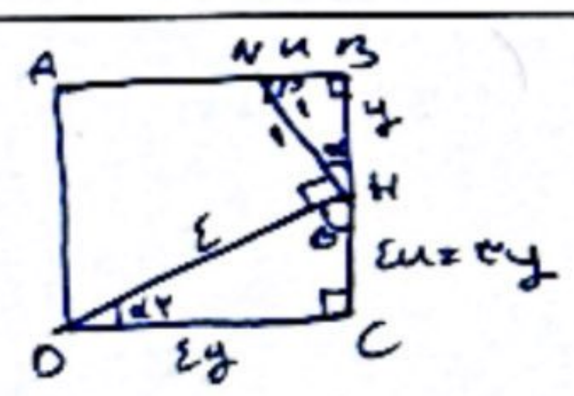

$\alpha + \beta + 90 = 180 \Rightarrow \alpha + \beta = 90$

مثلث NHB : $90 + \alpha + \hat{N}_1 = 180 \Rightarrow N_1 = 90 - \alpha = \beta$

مثلث HCO : $90 + \beta + \hat{O}_2 = 180 \Rightarrow \hat{O}_2 = 90 - \beta = \alpha$

$\Rightarrow \hat{B} = \hat{C} , \hat{H}_1 = \hat{O}_2 = \alpha , \hat{H}_2 = \hat{N}_1 = \beta \Rightarrow N\hat{H}B \sim O\hat{H}C \Rightarrow \frac{BN}{CH} = \frac{BH}{CO} = \frac{1}{4}$ → $4BH = CO\ (BH = y)$ ، $4BN = CH\ (BN = x)$

مربع ABCD $\Rightarrow BC = CD \Rightarrow 4y = y + 4x \Rightarrow 4x = 3y$

مثلث OCH : $(OH)^2 = (CH)^2 + (CD)^2 \Rightarrow (OH)^2 = (3y)^2 + (4y)^2$

$\Rightarrow OH = 5y = 4 \Rightarrow y = \frac{4}{5} \Rightarrow CD = 4 \times \frac{4}{5} = \frac{16}{5}$

$S_{\text{مربع}} = \left(\frac{16}{5}\right)^2 = 10.24$ ← جواب

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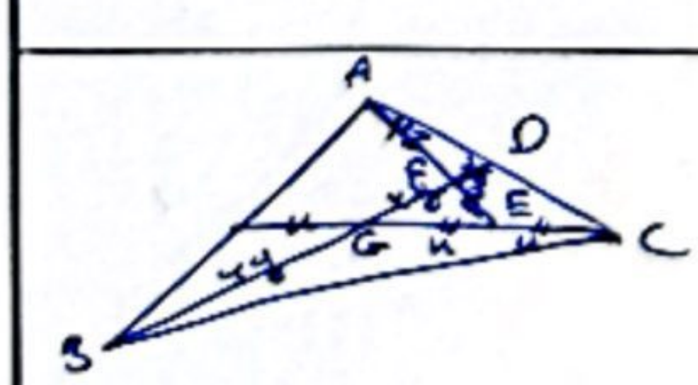

$\frac{BD}{FD} = ?$ ، $GE = EC$ ، G مرکز ثقل

G مرکز ثقل ⇐ G محل برخورد میانه هاست

⇐ CM میانه AB و BD میانه AC است

در مثلث میانه ها یکدیگر را به نسبت ۲ به ۱ قطع می کنند $BG = 2GD = 2 \times 3y = 6y$

در مثلث AGC : DG میانه AC و AE میانه GC است ⇐ $2y = GF = 2FD$

$\frac{BD}{FD} = \frac{9y}{y} = 9$ ← جواب