نام و نام خانوادگی: آیلین عظیمی ۲۰ تکلیف: ۲۳ کلاس: ۱۱ تجربی یازدهم دختر

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$\log_n m = a$

$\log_{mn} m^2 n = b \rightarrow \log_{mn} m^2 n = \dfrac{\log_n m^2 n}{\log_n mn} = \dfrac{\log_n m^2 + \log_n n}{\log_n m + \log_n n} = \dfrac{2a+1}{a+1} = b = \dfrac{a+a+1}{a+1} = \dfrac{a}{a+1} + 1 = b$

$[b] = ?$

$\rightarrow [b] = [\dfrac{a}{a+1}] + 1 = 1$

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الف) $\sqrt{\dfrac{x}{\log_{\frac{1}{2}} x}} \rightarrow$ ① $x > 0$ ② $\dfrac{x}{\log_{\frac{1}{2}} x} \geq 0$ $\quad \begin{array}{ccccc} & 0 & & 1 & \\ - & & + & & - \end{array} \rightarrow x \in [0,1)$

$\xrightarrow{① \cap ②} \boxed{D_f : (0,1)}$

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ب) $\dfrac{\log_4 (x^2-x-2)}{\sqrt{x^2-1}+1}$ ① $x^2-x-2>0 \rightarrow (x-2)(x+1)>0$ $\quad \begin{array}{ccccc} & -1 & & 2 & \\ + & & - & & + \end{array} \rightarrow x \in (-\infty,-1) \cup (2,+\infty)$

② $x^2-1 \geq 0 \rightarrow x^2 \geq 1 \rightarrow x \geq 1,\ x \leq -1$

$\xrightarrow{① \cap ②} \rightarrow x \in \boxed{(-\infty,-1) \cup (2,+\infty) : D_f}$

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$2\log_x a + \log_a \sqrt{x} = 2 \rightarrow 2\log_x a + \frac{1}{2}(\log_a x) = 2\log_x a + \dfrac{1}{2\log_x a} = 2$

$\log_a x \rightarrow \dfrac{1}{\log_x a}$

$x = 9$

$a = ?$

$\rightarrow 2\log_x a = 1 \rightarrow 2\log_9 a = 1 \rightarrow \log_9 a = \frac{1}{2} \rightarrow a = (9)^{\frac{1}{2}} = 3$

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$\log 2 = 0/3$

$\log 3 = 0/4$

$\log \frac{10}{5} = 0/3$

$1 - \log 5 = 0/3 \rightarrow \log 5 = 0/7$

$(\log \frac{5}{3})x^2 + (\log 9)x - \log 15 = 0$

$(\log 5 - \log 3)x^2 + (2\log 3)x - (\log 3 + \log 5) = 0$

$(0/7 - 0/4)x^2 + 0/8x - (0/4 + 0/7) = 0$

$0/3x^2 + 0/8x - 1/1 = 0$

$3x^2 + 8x - 11 = 0 \rightarrow x^2 + 8x - 33 = 0$

$(x+11)(x-\frac{3}{3})$

$(3x+11)(x-1)$

$x = 1,\ \frac{-11}{3}$

اختلاف: $1 + \frac{11}{3} = \frac{14}{3}$

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$\log_2 7 = 2/8$

$\log_5 2 = 0/5$

$\log_{14} 10 = ?$

$\log_{14} 10 = \dfrac{\log_5 10}{\log_5 14} = \dfrac{(\log_5 2) + 1}{(\log_5 2) + \log_5 7} = \dfrac{1/5}{0/5 + \log_5 7} \rightsquigarrow \log_5 7 = \dfrac{\log_2 7}{\log_2 5} = \dfrac{2/8}{\frac{1}{\log_5 2}} = \dfrac{2/8}{2} = 1/4$

$= \dfrac{1/5}{0/5 + 1/4} = \dfrac{1/5}{1/9} = \dfrac{15}{19}$

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$\log_3^5 = 1/5$

$\log_2^3 = 1/6$

$\log_{15}^6 = ?$

$$\log_{15}^6 = \log_{15}^2 + \log_{15}^3 = \frac{1}{\log_2^{15}} + \frac{1}{\log_3^{15}} = \frac{1}{\underset{1/6}{\log_2^3}+\log_2^5} + \frac{1}{\underset{1/5}{\log_3^5}+1}$$

$$\rightsquigarrow \log_2^5 = \frac{\log_3^5 \to 1/5}{\log_3^2} = \frac{1}{\log_2^3} = \frac{1}{1/6} = \frac{5}{8} = 2/4$$

$$= \frac{1}{1/6+2/4} + \frac{1}{1/5+1} = \frac{1}{4} + \frac{2}{5} = \frac{13}{20}$$

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$\log_8^{18} = m \rightarrow \log_8^{\overset{9\times 2}{18}} = \underbrace{\log_8^9}_{\log_{2^3}^{3^2}} + \log_8^{\cancel{2}\,\frac{1}{3}} = \frac{2}{3}\log_2^3 + \frac{1}{3} = m \rightarrow \log_2^3 = \frac{m-\frac{1}{3}}{\frac{2}{3}} = \frac{\frac{3m-1}{3}}{\frac{2}{3}} = \frac{3m-1}{2}$

$\log_4^{12} = ?$

$$\log_4^{\overset{4\times 3}{12}} = 1+\log_{2^2}^3 = 1+\frac{1}{2}\log_2^3 = 1+\frac{1}{2}\left(\frac{3m-1}{2}\right) = 1+\frac{3m-1}{4} = \frac{3m-1+4}{4}$$

$$= \boxed{\frac{3m+3}{4}}$$

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$(0/4)^{2x-1} = \left(\frac{125}{8}\right)^{x^2} \rightarrow \left(\frac{125}{8}\right)^{x^2} = \left(\frac{8}{125}\right)^{-x^2} = \left(\frac{2}{5}\right)^{-3x^2} = \left(\frac{2}{5}\right)^{2x-1} \rightarrow 2x-1 = -3x^2$

$3x^2+2x-1=0$

$a+c=b \rightarrow x=-1$ غ ق ق $\leftarrow 9x+1>0$

$x=\frac{1}{3}$

$\log_8^{9x+1} \xrightarrow{x=\frac{1}{3}} \log_8^4 = a \rightarrow 4 = 8^a$

$2^2 = 2^{3a} \rightarrow 2 = 3a \rightarrow \boxed{a=\frac{2}{3}}$

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$\log_2^3 = a$

$\log_8^b = \frac{2}{3}(a+1)$

$\log_8^b = \frac{2}{3}(\log_2^3 + \overset{\log_2^2}{\cancel{1}}) = \frac{2}{3}(\log_2^6) \rightsquigarrow \log_8^b = \frac{1}{3}\log_2^b$

$\rightsquigarrow \cancel{\tfrac{1}{3}}\log_2^b = \cancel{\tfrac{2}{3}}\,\log_2^6 \rightarrow \log_2^b = 2\log_2^6 = \log_2^{36}$

$b=36$

$\log(3b-8) = ? \xrightarrow{b=36} \log 100 = 2$

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$-4ax^2+bx+\frac{c}{2}=0 \rightarrow \log_{10}^4 = \frac{1}{\alpha+\beta} = \frac{1}{\log_4^{10}} \rightarrow \alpha+\beta = \log_4^{10}$ $\quad \alpha\beta = \frac{\frac{c}{2}}{-4a} = \frac{-c}{8a}$

$a = \frac{b+c}{2}$

$\left(\frac{1}{\sqrt[8]{2}}\right)^{\frac{c}{a}} = ?$

$$\rightarrow \left(\frac{1}{2^{\frac{1}{8}}}\right)^{\frac{c}{a}} = \left(\frac{2^0}{2^{\frac{1}{8}}}\right)^{\frac{c}{a}} = \left(2^{-\frac{1}{8}}\right)^{\frac{c}{a}} = 2^{\overset{\alpha\beta}{\frac{-c}{8a}}} = 2^{\alpha\beta} = ?$$

راه ①:

$$\xrightarrow{\text{از راه}} \alpha+\beta = \log_4^{10} = \log_4^{\cancel{2}} + \log_4^5 = \frac{1}{2} + \log_4^5 \rightarrow \alpha\beta = \left(\frac{1}{2}\right)(\log_4^5)$$

$$\rightarrow 2^{\alpha\beta} = 2^{\frac{\log_4^5}{2}} = \left(2^{\frac{1}{2}}\right)^{\log_4^5} = (\sqrt{2})^{\log_4^5} = (5)^{\frac{1}{4}} = \sqrt[4]{5}$$

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$$\log_{10}^4 = \frac{1}{\alpha+\beta} = \frac{1}{\frac{-b}{-4a}} = \frac{4a}{b} = \log_{10}^4$$

راه ②:

$$b = \frac{4a}{\log_{10}^4}$$

$$2a = b+c \rightarrow c = 2a-b = 2a - \frac{4a}{\log_{10}^4} = a\left(2-\frac{4}{\log_{10}^4}\right) = c$$

$$\frac{c}{a} = 2-\frac{4}{\log_{10}^4}$$

$$\rightarrow \left(\frac{1}{\sqrt[8]{2}}\right)^{\frac{c}{a}} = 2^{\frac{-c}{8a}} = \left(2^{-\frac{1}{8}}\right)^{2-\frac{4}{\log_{10}^4}}$$

$$= 2^{-\frac{1}{4}+\frac{1}{2\log_{10}^4}} = \frac{\cancel{\left(\frac{1}{2}\right)^{\frac{1}{4}}}}{2^{-\frac{1}{4}}} \times 2^{\frac{1}{2\log_{10}^4}\,\log_4^{10}} = 2^{-\frac{1}{4}} \times (\sqrt{2})^{\log_4^{10}} = \sqrt[4]{5}$$