شهرزاد شوشتری

(۱۶)

$\log_n m = a$, $\log_{mn} m^2 n = b \to 2\log_{mn} m + \log_{mn} n \to \frac{2}{\log_m m + \frac{1}{\log_n m}} + \frac{1}{\log_n m + \log_n n}$

$\frac{2}{\log_m mn} \to \frac{2}{\log_m m + \frac{1}{\log_n m}}$

$\to \frac{2}{1+\frac{1}{a}} + \frac{1}{a+1} \to \frac{2a}{a+1} + \frac{1}{a+1} = \frac{2a+1}{a+1} \to 1 + \frac{a}{a+1} \to \left[1 + \frac{a}{a+1}\right] = 1$; $1+\frac{1}{a} = \frac{a+1}{a}$

(۲)

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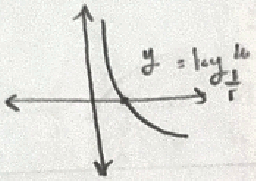
۲)

$\frac{x}{\log_{\frac{1}{2}} x} \geq 0 \to \log_2 x < 0 \to x < 2^0 = 1 \to \begin{cases} x>0 \\ x<1 \end{cases} \to D = (0,1)$

$y = \sqrt{\frac{x}{\log_{\frac{1}{2}} x}} \to \frac{x}{\log_{\frac{1}{2}} x} \geq 0$

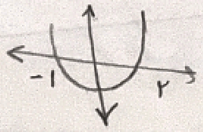


$\to D_f = \varnothing$

(۱)

$y = \frac{\log_4 (x^2 - x - 2)}{\sqrt{x^2-1}+1} \to x^2 - 1 \geq 0 \to x^2 \geq 1 \to x \geq 1 , x \leq -1$ ① / $x^2 - x - 2 > 0 \to \Delta > 0$

$D_f = (-\infty, -1) \cup (2, +\infty)$



$\to x < -1 , x > 2 \to \cap \to (-\infty,-1) \cup (2,+\infty)$

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۳)

$2\log_{\sqrt{3}} a + \log_a 3 = 2 \to \log_3 a + \frac{1}{\log_3 a} = 2$; $\log_3 a = t$

$t^2 + 1 = 2t \to t^2 - 2t + 1 = 0$

$(t-1)^2 = 0 \to t = 1 \to \log_a 3 = 1$ $\boxed{a = 3}$

(۲)

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۴)

$(\log 5 - \log 3)x^2 + (2\log 3)x - (\log 3 + \log 5)$; $\log_{10} 10 = \log 2 + \log 5 \to \log 5 = 0.7$

$(0.7 - 0.4)x^2 + 0.8x - (0.4 + 0.7)$

$0.3x^2 + 0.8x - 1.1 \xrightarrow{\times 10} 3x^2 + 8x - 11 \to x^2 + 8x - 33 \to (x+11)(x-3) = 0$

(۲)

$x = \frac{-11}{3}$, $x = \frac{3}{3} = 1$

$-\frac{11}{3} - 1 = -\frac{14}{3} \to \left|-\frac{14}{3}\right| = \frac{14}{3}$

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(۴

$\log_{15}^{6} = \frac{\log_2^6}{\log_2^{15}} \to \frac{\log_2^2 + \log_2^3}{\log_2^3 + \log_2^5} = \frac{1+1.6}{1.6+2.4} = 0.65$

$\log_3^5 \times \log_2^3 = \log_2^5 \to \log_3^5 \times \log_2^3 = 1.5 \times 1.6 = 2.4$

(۵

$\log_2^7 = 2.8 \quad \log_5^2 = 0.5 \quad \log_{14}^{10}$

$\log_5^2 = \frac{\log_{10}^2}{\log_{10}^{10} - \log_{10}^2} = \frac{\log 2}{1-\log 2} = \frac{1}{2} \to \log 2 = \frac{1}{3}$

$\log_2^7 = \frac{\log_{10}^7}{\frac{1}{3}} = \frac{28}{10} \to \log 7 = \frac{14}{15} \to \log_{14}^{10} = \frac{1}{\log_{10}^{14}} = \frac{1}{\log 2\times 7} = \frac{1}{\frac{1}{3}+\frac{14}{15}} = \frac{15}{19}$

(۷

$\log_8^{18} = m \to \frac{1}{3}\log_2^2 + \frac{2}{3}\log_2^3 = m \to \frac{2}{3}\log_2^3 = m - \frac{1}{3} \to \frac{1}{2}\log_2^3 = \frac{\frac{3m-1}{3}}{\frac{4}{3}}$

$\log_4^{12} = \frac{1}{2}\log_2^4 + \frac{1}{2}\log_2^3 = 1 + \frac{3m-1}{4} \to \frac{3m+3}{4} = \frac{3(m+1)}{4}$

(۸

$(0.16)^{2x-1} = \left(\frac{125}{8}\right)^{x^2} \to \log_8^{9x+1} = ?$

$\left(\frac{2}{5}\right)^{2x-1} = \left(\frac{5}{2}\right)^{3x^2} \to \left(\frac{5}{2}\right)^{1-2x} = \left(\frac{5}{2}\right)^{3x^2} \to 3x^2 = -2x+1 \to 3x^2+2x-1 = 0$

$3x^2+2x-1 = 0 \to x = -1$ غق

$x = \frac{1}{3} \qquad \log_8^4 = \frac{2}{3}\log_2^2 = \frac{2}{3}$

$\log_2^3 = a \quad \log_8^b = \frac{2}{3}(1+a) \to \log(3b-8)$

$\log_8^b = \frac{2}{3} + \frac{2}{3}\log_2^3 \to \log_8^b = \frac{2}{3} + \log_8^9 \to \log_8^b - \log_8^9 = \frac{2}{3} \to \log_8^{\frac{b}{9}} = \frac{2}{3}$

$8^{\frac{2}{3}} = \frac{b}{9} \to \frac{b}{9} = 4 \to b = 36 \qquad \log(3\times 36 - 8) = \log_{10}^{100} = 2$

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$-4ax^2 + bx + \frac{1}{2}c = 0 \Rightarrow \frac{1}{\alpha} + \frac{1}{\beta} = \log_{10}^{4}$ $\left(\frac{1}{\sqrt[4]{2}}\right)^{\frac{c}{a}}$

۵

$\frac{1}{\frac{b}{4a}} = \log 4 \Rightarrow \frac{4a}{b} = \log_{10}^{4}$ $\frac{a}{b} = \frac{\log 4}{4} = \frac{2\log 2}{4} = \frac{\log 2}{2}$

$2\left(\frac{\log 2}{2}\right) = 1 + \frac{c}{b} \Rightarrow \frac{c}{b} = -1 + \log 2$ $\frac{c}{b} = -1 + \log 2$, $\frac{a}{b} = \frac{\log 2}{2} \Rightarrow \frac{c}{a} = \frac{-1 + \log 2}{\frac{\log 2}{2}} = \frac{\log \frac{2}{10}}{\frac{1}{2}\log 2}$

$= \frac{\log \frac{1}{5}}{\frac{1}{2}\log 2} = \frac{\log \frac{1}{5}}{\log \sqrt{2}} = \log_{\sqrt{2}} \frac{1}{5} \Rightarrow \left(\frac{1}{\sqrt[4]{2}}\right)^{\frac{c}{a}} \Rightarrow \left(\frac{1}{\sqrt[4]{2}}\right)^{\log_{\sqrt{2}} \frac{1}{5}} \Rightarrow \frac{1}{5}^{\log_{\sqrt{2}} \frac{1}{\sqrt[4]{2}}}$

$\left(\frac{1}{5}\right)^{-\frac{1}{4}} \Rightarrow 5^{\frac{1}{4}} \Rightarrow \sqrt[4]{5}$