تکلیف شماره ۲۳ — دنیا جمشیدی — ۲۰ — یازدهم ششم

① $\log_n m = a \qquad \log_{mn} m^2 n = b$

$a > 0 \longrightarrow [b] = ?$

$m = n^a$

$\log_{mn} m^2 n = \log_{n^a \times n} (n^a)^2 \times n$ — ۶

$\log_{n^{a+1}} n^{2a+1} = \frac{2a+1}{a+1} \log_n n = [\;]$

$\frac{2a+1}{a+1} = \frac{a+1+a}{a+1} = 1 + \frac{a}{a+1} \longrightarrow$

$1 + [\frac{a}{a+1}] = 1$

② الف) $y = \sqrt{\frac{x}{\log \frac{x}{2}}}$ — $x \to 0$ = ریشه، $x \to 1$ = ریشه — $x > 0$ — ۶

$x > 1 \to \log \frac{x}{2} < 0 \qquad x < 1 \to \log \frac{x}{2} > 0$

$\begin{array}{c|c|c|c} & 0 & 1 & \\ - & + & - & \end{array} \longrightarrow D_f = (0, 1)$

ب) $\frac{\log_2 (x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$

$x^2 - x - 2 > 0 \qquad (x+1)(x-2) > 0$

$(-\infty, -1) \cup (2, +\infty)$

$\sqrt{x^2 - 1} \neq -1$ همواره برقرار ✓

$D_f = (-\infty, -1) \cup (2, +\infty)$

③ $2 \log_x a + \log_a \sqrt{x} = 2 \qquad x = 9$

$a = ? \qquad \frac{1}{2} \log_3 a$

$x = 9 \to 2 \log_9 a + \log_a 3 = 2$

$\log_3 a + \log_a 3 = 2 \qquad \log_3 a = t$ — ۶

$t + \frac{1}{t} = 2 \qquad \frac{t^2 + 1}{t} = 2$

$t^2 - 2t + 1 = 0 \qquad (t-1)^2 = 0 \qquad t = 1$

$\log_3 a = 1 \qquad a = 3$

s.a.m

④

$\log 2 \approx 0.3$ $\quad \log 3 \approx 0.4$ $\quad \log 10 = 1$

$(\log \frac{5}{3})x^2 + (\log 9)x - \log 10 = 0$

اصل ریشه ها؟

$\log \frac{5}{3} = \log 5 - \log 3$

$\log 5 = ?$ $\quad \log \frac{10}{2} = \log 10 - \log 2 = 1 - 0.3 = 0.7$

$0.7 - 0.4 = 0.3$

$\log 9 = \log 3^2 = 2\log 3 = 0.8$

$\log 10 = \log 3 \times 5 = \log 3 + \log 5 =$

$0.4 + 0.7 = 1.1$

$\Delta = 0.64 - 4(0.3)(-1.1) = 1.96$

$x_{1,2} = \frac{-(-1.2) \pm \sqrt{\Delta}}{2a}$

$\frac{\sqrt{1.96}}{0.3} = \frac{1.4}{0.3} = \frac{14}{3}$

⑤

$\log 2 = \frac{1}{3.4}$ $\quad \log 3 = \frac{1.6}{3.4}$ $\quad \log 5 = \log 10 - \log 2 = 1 - \log 2$

$2 - 2\log 2 = 3\log 3$ $\quad 2 - 2\log 2 = 4.8\log 2$

$\log \frac{3}{2} = \frac{\log 3}{\log 2} = 1.6$

$\log 15 = \log 6 + \log 5$

$\log 15 = \log 3 + \log 5$

$\frac{\log 2 + \log 3}{\log 3 + \log 5} = \frac{\frac{1.6+1}{3.4}}{\frac{0.6+3.4}{3.4}} = 0.4$

$\log 10 - \log 2 = 1 - \log 2$

$1.6\log 2 + 1 - \log 2 = 0.6\log 2 + 1$

⑥

$\log 3 = 1.6$

$\log 2 = 1.4$

$\log 5 = ?$

$\frac{\log 5}{\log 3} = \frac{3}{2}$ $\quad \frac{\log 10 - \log 2}{\log 3} = \frac{3}{2}$

⑦

$\log_{12} 8 = m$ $\quad \log_{12} 8 = \log_{12} 2^3 + \log_{12} 3^3$

$\log_{12} 4 = ?$ $\quad \frac{2}{3}\log_{12} 2 + \frac{1}{3}\log_{12} 3$

$\log_{12} 2^2 + \log_{12} 3^2 = \frac{2}{3}\log_{12} 3 + \frac{1}{3}\log_{12} 2 = m$

$\frac{2}{3}\log_{12} 3 = m - \frac{1}{3}$ $\quad \log_{12} 3 = \frac{3m-1}{2}$

$\log_{12} 8 = \frac{2}{3} + \frac{3m-1}{6}$ $\quad \frac{3}{4}(m+1)$

$\log_8 4 = \log_{2^3} 2^2 = \frac{2}{3}\log_2 2 = \frac{2}{3}$

جواب $= \log_8^{12} 4 = \frac{\log_8^{12}}{\log_8^4} = \frac{\frac{2}{3} + \frac{3m-1}{6}}{\frac{2}{3}} =$

③

$\log_2 7 = 2.8$ $\quad \log \frac{2}{5} = \frac{1}{2}$

$\log \frac{2}{5} = 0.5$ $\quad \frac{\log 2}{\log 5} = \frac{1}{2}$ $\quad \frac{\log 2}{\log 10 - \log 2} = \frac{1}{2}$

$\log 10 = ?$ $\quad \frac{\log 2}{\log 10 - 2} = \frac{1}{2}$

$2\log 2 = 1 - \log 2 \rightarrow \log 2 = \frac{1}{3}$

$\log_2 7 = \frac{\log 7}{\log 2} = 2.8 \rightarrow \log 7 = \frac{1}{3} \times 2.8 =$

$\log_{14} 10 = \frac{\log 10}{\log 14} = \frac{1}{\log 2 + \log 7} = \frac{1}{\frac{1}{3} + \frac{14}{15}} = \frac{15}{19}$

s.a.m

۸) $(0.4)^{2x-1} = \left(\frac{125}{8}\right)^{x^2}$

$\log_8^{9x+1} = ?$

$\log_{0.4}\left(\frac{125}{8}\right)^{x^2} = 2x-1$

$x^2\left(\log_{0.4}\frac{125}{8}\right) = 2x-1$

$\log\frac{125}{8} \to \log 125 - \log 8 = \log 5^3 - \log 2^3$

$\log\frac{2}{5} = -\log\frac{5}{2} = -3(\log 5 - \log 2)$

$\hookrightarrow \log 2 - \log 5$

$\frac{3(\log 5 - \log 2)}{(\log 2 - \log 5)} = -3$

$\hookrightarrow -3x^2 = 2x-1 \qquad 3x^2+2x-1=0 \qquad a+c=b$

$\left\{\begin{matrix} -1 \\ \frac{1}{3}\end{matrix}\right.$

$\log_8^{9x+1} = \begin{cases} x=-1 \to \log_8^{-8} \\ x=\frac{1}{3} \to \log_8^4 = \log_{2^3}^{2^2} = \end{cases}$

$\frac{2}{3}\log_2^2 = \frac{2}{3}$

۹) $\log_{20}^3 = a$

$\log_8^b = \frac{2}{3}(1+a)$

$\log(3b-8) = ?$

$\log_8^b = \frac{2}{3} + \frac{2}{3}a$

$= \log_8^{3^2} = \log_8^9$

$\log_8^b = \left(\frac{2}{3}\right) + \log_8^9$

$\hookrightarrow = \log_8^4 = \log_{2^3}^{2^2} = \frac{2}{3}$

$\log_8^b = \log_8^4 + \log_8^9 = \log_8^{36}$

$b = 36 \qquad \log_{10}^{(100)} = 2$

۱۰) $-4ax^2 + bx + \frac{1}{2}c = 0$

$\frac{4a}{b} = \log 4 \qquad \frac{b+c}{2} = a$

$\frac{a}{b} = \frac{\log 4}{4} = \frac{2\log 2}{4} = \frac{\log 2}{2}$

$4a = b+c \xrightarrow{\div b} \frac{4a}{b} = 1 + \frac{c}{b}$

$\log 2 = 1 + \frac{c}{b}$

$\frac{c}{b} = \log 2 - 1$

s.a.m

ادامه سوال ۱۵ :

$$\begin{cases} \frac{c}{b} = -1 + \log 2 \\ \frac{a}{b} = \frac{\log 2}{2} \end{cases} \qquad \frac{c}{a} = \frac{\log 2 - 1}{\frac{\log 2}{2}}$$

$$\frac{c}{a} = \frac{\log 2 - \log 10}{\frac{1}{2}\log 2} \quad \text{و} \quad \log \frac{2}{10} = \log \frac{1}{5}$$

$$\log 2^{\frac{1}{2}} = \log \sqrt{2}$$

$$\frac{c}{a} = \frac{\log \frac{1}{5}}{\log \sqrt{2}} = \log_{\sqrt{2}} \frac{1}{5}$$

$$\left(\frac{1}{\sqrt[8]{2}}\right)^{\frac{c}{a}} = \left(\frac{1}{\sqrt[8]{2}}\right)^{\log_{\sqrt{2}} \frac{1}{5}}$$

$$\log_{\sqrt{2}} \frac{1}{5} \rightarrow \log_{2^{\frac{1}{2}}} 2^{-\frac{1}{8}} = \frac{-\frac{1}{8}}{\frac{1}{2}} = -\frac{1}{4}$$

$$-\frac{1}{4} = \left(\frac{1}{5}\right)^{-\frac{1}{4}} = 5^{\frac{1}{4}} = \sqrt[4]{5}$$

s.a.m