عرفان کاظمی — یازدهم — تکلیف شماره

۲۰

۱)

$\log_n^m = a \to n^a = m \qquad \log_{mn}^{m^2n} = b \Rightarrow \log_{n^{a+1}}^{n^{2a+1}} = b \to \frac{2a+1}{a+1}\log_n^n = b$ ۵

$\Rightarrow b = \frac{2a+1}{a+1} \to [b] = \left[\frac{2a+1}{a+1}\right] = \left[\frac{a+a+1}{a+1}\right]$

$= \left[\frac{a}{a+1}+1\right] = \left[\frac{a}{a+1}\right]+1 = \boxed{1}$

۲)

الف) $y = \sqrt{\frac{x}{\log_{\frac{1}{2}}^x}} \qquad \frac{x}{\log_{\frac{1}{2}}^x} \geq 0 \qquad \log_{\frac{1}{2}}^x > 0 \to x < 1$

$x > 0$

$\xrightarrow{\cap} 0 < x < 1 \to (0,1)$ ۵

ب) $y = \frac{\log_2^{(x^2-x-2)}}{\sqrt{x^2-1}+1} \to (x-2)(x+1) > 0 \qquad \frac{-1 \quad 2}{+|-|+} \qquad (-\infty,-1) \cup (2,+\infty)$

$\to x^2-1 \geq 0 \qquad (x-1)(x+1) \geq 0 \qquad \frac{-1 \quad 1}{+|-|+} \qquad (-\infty,-1] \cup [1,+\infty)$

$\xrightarrow{\cap} \boxed{(-\infty,-1) \cup (2,+\infty)}$

۳)

$2\log_x^a + \log_a^{\sqrt{x}} = 2 \to 2\log_9^a + \log_a^3 = 2$

$\log_3^a + \log_a^3 = 2 \to \underbrace{\log_3^a}_{t} + \frac{1}{\log_3^a} = 2$ ۵

$t + \frac{1}{t} = 2 \to t^2 - 2t + 1 = 0 \to (t-1)^2 = 0 \to t = 1 \to \log_3^a = 1 \to \boxed{a = 3}$

۴)

$\log 2 \simeq 0/3 \qquad \log 3 \simeq 0/4$

$\left[\log^{\frac{5}{3}}\right]x^2 + (\log 9)x - \log 15 = 0 \longrightarrow 0/3x^2 + 0/8x - 1/1 = 0$ ۵

$3x^2 + 8x - 11 = 0 \to \begin{cases} x = -\frac{11}{3} \\ x = 1 \end{cases}$

$\log^5 - \log^3 = \log 10 - \log 2 - \log^3 = 1 - \underbrace{0/3 - 0/4}_{0/7} = 0/3 \qquad 1 - \left(-\frac{11}{3}\right) = \boxed{\frac{14}{3}}$

$\log 15 = \log 3 + \log\frac{10}{2} = \log 3 + \log 10 - \log 2 = 1/4 - 0/3 = 1/1$

(5

$\log_2 7 = 2{,}8 \qquad \log_5 2 = 0{,}5 \rightarrow \dfrac{\log 2}{\log 5} = \dfrac{1}{2} \rightarrow \dfrac{\log 2}{\log \frac{10}{2}} = \dfrac{1}{2} \rightarrow \dfrac{\log 2}{\log 10 - \log 2} = \dfrac{1}{2} \Rightarrow \dfrac{\log 2}{1-\log 2} = \dfrac{1}{2}$

$\log 2 = \dfrac{1}{3}$

$\log_{14} 10$

$= \dfrac{\log 7}{\log 2} = \dfrac{28}{10} \rightarrow \dfrac{\log 7}{\frac{1}{3}} = \dfrac{28}{10} \Rightarrow \log 7 = \dfrac{14}{15}$

$= \dfrac{1}{\log_{10} 14} = \dfrac{1}{\log 2 + \log 7} = \dfrac{1}{\frac{1}{3}+\frac{14}{15}} = \boxed{\dfrac{15}{19}}$

ص

(6

$\log_3 5 = 1{,}5 \qquad \log_2 3 = 1{,}6 \rightarrow \log_3 2 = \dfrac{10}{16} = \dfrac{5}{8}$

ص

$\log_{15} 6 = ? \rightarrow \dfrac{\log_3 6}{\log_3 15} = \dfrac{\log_3 2 + \log_3 3}{\log_3 3 + \log_3 5} = \dfrac{\frac{5}{8}+1}{\frac{3}{2}+1} = \boxed{0{,}65}$

(7

$\log_8 18 = m \qquad \log_4 12 = ?$

$\log_4 12 = \dfrac{\log_8 12}{\log_8 4}$

ص

$\log_8 12 = \log_8 4 + \log_8 3 = \dfrac{2}{3}\log_2 2 + \dfrac{1}{3}\log_2 3 = \dfrac{2}{3} + \dfrac{1}{3}\log_2 3$

$\log_8 4 = \dfrac{2}{3}\log_2 2 = \dfrac{2}{3}$

$\log_8 18 = m \Rightarrow \log_8 9 + \log_8 2 = m \Rightarrow \dfrac{2}{3}\log_2 3 + \dfrac{1}{3}\log_2 2 = m$

$\rightarrow \log_2 3 = \dfrac{3m-1}{2}$

$\dfrac{\frac{2}{3}+\frac{1}{3}\log_2 3}{\frac{2}{3}} = \dfrac{\frac{2}{3}+\frac{1}{3}\left(\frac{3m-1}{2}\right)}{\frac{2}{3}} = \dfrac{2+\left(\frac{3m-1}{2}\right)}{2} = \dfrac{4+3m-1}{4}$

$= \boxed{\dfrac{3}{4}(m+1)}$

(8

$(0{,}4)^{2n-1} = \left(\dfrac{125}{8}\right)^{n^2} \Rightarrow \left(\dfrac{2}{5}\right)^{2n-1} = \left(\dfrac{5}{2}\right)^{3n^2} \rightarrow \left(\dfrac{5}{2}\right)^{1-2n} = \left(\dfrac{5}{2}\right)^{3n^2}$

$\log_8 (9n+1) = ?$

$3n^2 = -2n+1 \rightarrow 3n^2+2n-1=0$

$n = -1$ غ ق ق

$n = \dfrac{1}{3}$

$\log_8 9\left(\frac{1}{3}\right)+1 = \log_8 4 = \dfrac{2}{3}\log_2 2 = \boxed{\dfrac{2}{3}}$

ص

(9

$\log_2^3 = a$

$\log_8^b = \frac{3}{2}(1+a)$

$\log(3b-8) = ?$

$\log_8^b = \frac{3}{2}(1+\log_2^3) = \frac{3}{2} + \frac{3}{2}\log_2^3$

$\Rightarrow \log_8^b - \log_8^9 = \frac{3}{2} \rightarrow \log_8^{\frac{b}{9}} = \frac{3}{2}$

$8^{\frac{3}{2}} = \frac{b}{9} \Rightarrow b = 36$

$\sqrt[3]{2^{12}} = 4$

$\log(108-8) = \log 100 = \boxed{2}$

(10

$-4ax^2+bx+\frac{1}{2}c=0 \longrightarrow \frac{1}{\frac{-b}{-4a}} = \log 4 \Rightarrow \frac{4a}{b} = \log 4$

$\frac{a}{b} = \frac{\log 4}{4} = \frac{2\log 2}{4} = \frac{\log 2}{2}$

$\frac{b+c}{2} = a \Rightarrow c = 2a-b \xrightarrow{\div a} \frac{c}{a} = 2 - \frac{b}{a} = 2 - \left(\frac{2}{\log 2}\right) = 2\left(1 - \frac{1}{\log 2}\right)$

$= 2(\underbrace{1 - \log_2^{10}}_{\log_2^2 - \log_2^{10}}) = 2(\log_2^{0.2})$

$\left(\frac{1}{\sqrt[8]{2}}\right)^{\frac{c}{a}} = \left(2^{-\frac{1}{8}}\right)^{\frac{c}{a}} = 2^{-\frac{1}{4}\log_2^{0.2}} = 2^{\log_{16}^5} \Rightarrow 5^{\log_{16}^2} = 5^{\frac{1}{4}\log_2^2} = \boxed{\sqrt[4]{5}}$