۶-

$\log_3^5 = 1.5$, $\log_2^3 = 1.6$, $\log_{15}^6 = ?$

$\to \log 5 = 1.5\log 3$; $\frac{5}{8}\log 3 = \log 2$ (۲)

$\log_{15}^6 = \frac{\log 3 + \log 2}{\log 5 + \log 3} = \frac{1+\frac{5}{8}}{1.5+1} = \boxed{\frac{13}{20}}$

۷-

$\log_8^{18} = m$, $\log_4^{12} = ?$

$\frac{\log 2 + 2\log 3}{3\log 2} = m \to 2\log 3 = (3m-1)\log 2$ (۲)

$\log_4^{12} = 1 + \frac{\log 3}{2\log 2} = 1 + \frac{3m-1}{4} = \boxed{\frac{3m+3}{4}}$

۸-

$(0.16)^{2x-1} = \left(\frac{125}{8}\right)^{x^2}$, $\log_8^{(9x+1)} = ?$

✓ $(x > -\frac{1}{9}) \leftarrow 9x+1 > 0$ (۲)

$\left(\frac{2}{5}\right)^{2x-1} = \left(\frac{5}{2}\right)^{3x^2} \to 1-2x = 3x^2 \to (3x-1)(x+1) = 0 \to x = -1$ (غ‌ق) , $\boxed{\frac{1}{3}}$

$\log_8^{(9\times\frac{1}{3}+1)} = \log_8^4 \Rightarrow 8^O = 4 \to 2^{3O} = 2^2 \to \boxed{O = \frac{2}{3}}$ ✓

۹-

$\log_2^3 = a$, $\log_8^b = \frac{2}{3}(1+a)$, $\log(3b-8) = ?$

$\frac{\log 3}{\log 2} = a$ ، $\log_8^b = \frac{2}{3}(1+a) \to \frac{\log b}{3\log 2} = \frac{2}{3}(1+a) \to \log b = 2(1+a)\log 2$ (۲)

$\to \log b = 2\log 2 + 2\log 3 \to \log b = \log 36 \to b = 36$

$\log(100) = \boxed{2}$

۱۰-

$-4ax^2 + bx + \frac{1}{2}c = 0$ ، عکس مجموع ریشه‌ها $= \log^4$ ، واسطه حسابی b و c $= a$ ، $\left(\frac{1}{\sqrt{2}}\right)^{\frac{c}{a}} = ?$

$\frac{1}{\alpha+\beta} = \frac{1}{S} = \frac{1}{\frac{b}{4a}} = \frac{4a}{b} = \log 4 \to \frac{4a}{2a-c} = \log 4$ (۲)

$\frac{2a-c}{4a} = \log_4^{10} \to \frac{1}{2} - \frac{c}{4a} = \log_4^{10} \to \frac{1}{2} - \log_4^{10} = \frac{c}{4a} \to \log_4^{\frac{2}{10}} = \frac{c}{4a}$

$\log_4^{\left(\frac{1}{5}\right)^4} = \frac{c}{a} \to \left(\frac{1}{\sqrt{2}}\right)^{\frac{c}{a}} = 2^{-\frac{1}{2}\times\log_4^{5^{-4}}} = 2^{\log_4^{5^2}} = \sqrt{5}^{\log_4^2} = \boxed{\sqrt[4]{5}}$

زهرا الله‌دادی ، یازدهم دختری ... نام ...

۲۰

۱-

$\log_n m = a$ ، $\log_{mn} m^2 n = b$ ، $a > 0$ ، $[b] = ?$

$\dfrac{\log m}{\log n} = a$ → $1 + \dfrac{\log m}{\log m + \log n} = b$ → $1 + \dfrac{a}{a+1} = b$ → $[b] = 1$

$0 < \dfrac{a}{a+1} < 1$

۲-

الف) $y = \sqrt{\dfrac{x}{\log_{\frac{1}{2}} x}}$ ، $x > 0$ ، $x \neq 1$ ، $\dfrac{x}{\log_{\frac{1}{2}} x} \geq 0$ → $D_y = (0, 1)$

ب) $y = \dfrac{\log_4 (x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$ → $x^2 - x - 2 > 0$ → $(x-2)(x+1) > 0$

$x^2 - 1 \geq 0$ → $x^2 \geq 1$ → $x \geq 1$, $x \leq -1$

$D_y = (-\infty, -1) \cup (2, +\infty)$ $\sqrt{x^2 - 1} + 1 \neq 0$ → $\sqrt{x^2 - 1} \neq -1$ ✓

۳-

$2 \log_x a + \log_a \sqrt{x} = 2$ ، $x = 9$ ، $a = ?$

$\frac{1}{2} \log_a x = \dfrac{1}{2 \log_x a}$ → ($x = 9$) $2 \log_9 a + \dfrac{1}{2 \log_9 a} = 2$ ← عددی که جمع آن با معکوسش ۲ بشود ۱ است.

→ $2 \log_9 a = 1$ → $a^2 = 9$ → $a = 3$

۴-

$\log 2 \simeq 0.3$ ، $\log 3 \simeq 0.48$ → $\log 5 \simeq 0.7$

$\left[\log \frac{5}{3}\right] x^2 + (\log 9) x - \log 15 = 0$

0.2 ، 0.96 ، $0.48 + 0.7 = 1.18$

$2x^2 + 9x - 11 = 0$ → $(2x + 11)(x - 1) = 0$ → $x = -\frac{11}{2}$, $x = 1$

اختلاف ریشه‌ها : $-\frac{11}{2} - 1 = -\frac{13}{2}$ → اختلاف ریشه‌ها $= \frac{13}{2}$ ✓

۵-

$\log_2 7 = 2.8$ ، $\log_5 2 = 0.5$ ، $\log_{14} 10 = ?$

→ $\log 7 = 2.8 \log 2$ → $2 \log 2 = \log 5$

$\log_{14} 10 = \dfrac{\log 2 + \log 5}{\log 2 + \log 7} = \dfrac{2 + 1}{1 + 2.8} = \dfrac{3}{3.8}$