روزانه صدرالعلمایی — تکلیف ۳ — یازدهم دختر C

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$\log_n^m = a \qquad \log_{mn}^{m^2 n} = b \qquad a>0 \qquad [b] = ?$

$\dfrac{\log_n^{m^2 n}}{\log_n^{mn}} = \dfrac{\log_n^{m^2} + \log_n^{n}}{\log_n^m + \log_n^{n}} = \dfrac{2a+1}{a+1} \xrightarrow{a+1+a} \left[a + \dfrac{a}{a+1}\right] = 1$

بین صفر و یک

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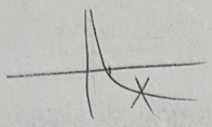
$y = \sqrt{\dfrac{x}{\log_{\frac{1}{2}} x}} \geq 0 \longrightarrow x>0$



چون x همیشه باید بزرگتر صفر باشد پس صورت همیشه مثبت است

پس باید مخرج هم مثبت و غیرصفر باشد پس $x \neq 1$ است و

x های بزرگتر از ۱ جواب را منفی می‌کنند

$\boxed{D_f = (0,1)}$

ب) $y = \dfrac{\log_4^{x^2-x-2}}{\sqrt{x^2-1}+1}$

① $x^2-1 \geq 0 \quad x^2 \geq 1 \longrightarrow x \geq 1 \quad x \leq -1$

② $x^2-x-2>0 \qquad (x-2)(x+1)>0$

	-1		2	
$+$		$-$		$+$

$\boxed{①,② \rightarrow (-\infty,-1) \cup (2,+\infty)}$

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$2\log_x^a + \log_a^{\sqrt{x}} = 2 \qquad x=9 \quad a=?$

$\longrightarrow 2\underbrace{\log_9^a}_{t} + \frac{1}{2}\log_a^9 = 2 \rightarrow 2t + \frac{1}{2}\times\frac{1}{t} = 2 \xrightarrow{\times t}$

$2t^2 - 2t + \frac{1}{2} = 0 \xrightarrow{\times 2}$

$4t^2 - 4t + 1 = 0 \rightarrow (2t-1)^2 = 0 \rightarrow \frac{1}{2}$

$\log_9^a = \frac{1}{2} \qquad \boxed{a = \sqrt{9} = 3} \longleftarrow$

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$\left(\log\frac{5}{3}\right)x^2 + \left(\log 9\right)x - \log 15 = 0 \qquad \log 2 = 0{,}3 \qquad \log 3 = 0{,}4$

$\underbrace{2\log 3} \qquad \rightarrow \log 5 + \log 3 \qquad \rightarrow \log 10 - \log 2 = \log 5$

$0{,}3x^2 + 0{,}8x - 1{,}1 = 0 \qquad 0{,}7 + 0{,}4 \qquad 1 - 0{,}3 = 0{,}7$

$\log\frac{5}{3} = \log 5 - \log 3 = 0{,}7 - 0{,}4 = 0{,}3$

$3x^2 + 8x - 11 = 0 \rightarrow x^2 + 8x - 11\times 3 = 0$

$(x+11)(x-3) = 0 \quad \rightarrow -\frac{11}{3} \quad \rightarrow \frac{3}{3} \longrightarrow \left|-\frac{11}{3} - \frac{3}{3}\right| = \boxed{\frac{14}{3}}$

$\dfrac{1}{\log_5^2} = \log_2^5 = 2$

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$\log_{14}^{10} = ? \qquad \log_5^2 = 0{,}5 \qquad \log_2^7 = 2{,}8$

$\dfrac{\log_2^{10}}{\log_2^{14}} = \dfrac{\cancel{\log_2^2} + \cancel{\log_2^5}}{\cancel{\log_2^7} + \cancel{\log_2^2}} = \dfrac{3}{3{,}8} = \dfrac{30}{38} = \boxed{\dfrac{15}{19}}$

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$\log_{15}^{6} = ?$ $\log_2^3 = 1.6$ $\log_3^5 = 1.5 \longrightarrow \frac{\log_2^5}{\log_2^3} = \frac{3}{2}$ 1.6 $\log_2^5 = 2.4$ —6

$$\frac{\log_2^6}{\log_2^{15}} = \frac{\log_2^2 + \log_2^3}{\log_2^3 + \log_2^5} = \frac{1+1.6}{1.6+2.4} = \frac{2.6}{4} = \frac{26}{40} = \frac{13}{20} = \boxed{0.65}$$

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$\log_8^{18} = m$ $\log_4^{12} = ? = \log_4^{2^2} + \log_4^3 = 1 + \log_4^3 = 1 + m - \frac{1}{4} = \boxed{\frac{3}{4} + m}$ —7

$$\frac{\log_4^{18}}{\log_4^{8}} = \frac{\log_4^{18}}{2} = m \longrightarrow 2m = \log_4^{3^2} + \underset{\frac{1}{2}}{\log_4^2} \longrightarrow \frac{2m - \frac{1}{2}}{2} = \log_4^3 = m - \frac{1}{4}$$

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$(0.4)^{2x-1} = \left(\frac{125}{8}\right)^{x^2}$ $\log_x^{9x+1} \rightarrow \log_8^4 = \log_{2^3}^{2^2} \boxed{= \frac{2}{3}}$ —8

$$\left(\frac{2}{5}\right)^{2x-1} = \left(\frac{5}{2}\right)^{3x^2} \longrightarrow -2x+1 = 3x^2 \rightarrow 3x^2 + 2x - 1 = 0 \quad x^2 + 2x - 3 \quad (x+3)(x-1) = 0$$

غ‌ق‌ق $-1 = \frac{-3}{3}$ $\frac{1}{3}$ ✓

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—9

$\log_2^3 = a$ $\log_8^b = \frac{2}{3}(1+a)$ $\log^{3b-8} = ? = \log^{3\times 36 - 8 = 100}$ $= (2)$

$$\cancel{\frac{1}{3}}\log_2^b = \frac{2}{\cancel{3}}(1+a)$$

$$\log_2^b = \underset{\log_2^4}{2} + \underbrace{2(\log_2^3)}_{\log_2^9} = \log_2^{36}$$

—10

$-4ax^2 + bx + \frac{1}{2}c = 0$ $\log^4 = \frac{1}{x_1 + x_2}$ $\frac{b+c}{2} = a$

$\left(\frac{1}{\sqrt[8]{2}}\right)^{\frac{c}{a}}$ $2^{\frac{1}{8}}$ $\frac{-b}{-4a} \rightarrow \frac{4a}{b} = \log^4 \rightarrow \frac{2a}{b} = \log_{10}^2 \rightarrow b = \frac{2a}{\log^2}$

$$c = 2a - b \xrightarrow{\div a} \frac{c}{a} = 2 - \frac{b}{a} = 2 - \left(\frac{2}{\log_{10}^2}\right) = 2 - 2\log_2^{10}$$

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$$2^{-\frac{1}{8}(2 - 2\log_2^{10})} = 2^{-\frac{1}{4}(\underset{\log_2^2}{1} - \log_2^{10})} = 2^{-\frac{1}{4}(\log_2^{0.2})} = 2^{\log_2^{5^{\frac{1}{4}}}} \rightarrow \boxed{5^{\frac{1}{4}} = \sqrt[4]{5}}$$

$$\frac{\log^{18}}{\log^8} \Rightarrow \frac{\log^3}{\log^2} = \frac{3m-1}{2} \qquad \log_4^{12} = 1 + \cancel{\log_4^3} \rightarrow \frac{\log^3}{2\log^2} = 1 + \frac{1}{4}\left(\frac{3m-1}{2}\right) = \frac{3m+7}{8}$$