
-۱

$\log_n m = a \to m = n^a$

$\log_{(n^a)\times n} (n^a)^2 \times n = b \Rightarrow \frac{2a+1}{a+1} \log_n n = b \to \frac{2a+1}{a+1} = b \xrightarrow{\text{تفکیک}} \underbrace{1 + \overbrace{\frac{a}{a+1}}^{\text{کوچکتر از یک}}}_{1/\ldots} = b \Rightarrow [b] = 1$

(۲)

-۲

الف) $\frac{x}{\log_{\frac{1}{2}} x} \geq 0$, $x > 0$

$\log_{\frac{1}{2}} x = 0 \to x = 1$

$-\infty \quad 0 \quad 1 \quad +\infty$: $- \quad + \quad -$ $\Rightarrow D_f = (0,1)$

(۲)

ب) $x^2 - x - 2 > 0 \to$ $-\infty \quad -1 \quad 2 \quad +\infty$: $+ \quad - \quad +$ $\to (-\infty,-1) \cup (2,+\infty)$

$x^2 - 1 \geq 0 \to$ $-\infty \quad -1 \quad 1 \quad +\infty$: $+ \quad - \quad +$ $\to (-\infty,-1] \cup [1,+\infty)$

$\xrightarrow{\cap} (-\infty,-1) \cup (2,+\infty)$

-۳

$\underbrace{2\log_9 a}_{\log_3 a} + \log_a 3 = 2 \Rightarrow \log_3 a + \frac{1}{\log_3 a} = 2 \xrightarrow[\times t]{\log_3 a = t} t^2 - 2t + 1 = 0 \to 1 = t \to$

$\log_3 a = 1 \to a = 3^1$

(۲)

-۴

$\log 5 = \log \frac{10}{2} = \underbrace{\log 10}_{1} - \underbrace{\log 2}_{0/3} = 0/7$

$(\underbrace{\log 5 - \log 3}_{0/7 - 0/4 = 0/3})x^2 + (\underbrace{2\log 3}_{0/8})x - (\underbrace{\log 5 + \log 3}_{0/7 + 0/4 = 1/1}) = 0 \xrightarrow{\div 0/1} 3x^2 + 8x - 11 = 0$

(۱) $\qquad -\frac{11}{3}$

تفاضل $= 1 - (-\frac{11}{3}) = \frac{14}{3}$

-۵

$\log_{14} 10 = \frac{\log_2 10}{\log_2 14} = \frac{\log_2 5 + \log_2 2}{\log_2 7 + \log_2 2} = \frac{2+1}{2/8+1} = \frac{3}{3/8} = \frac{30}{38}$

$\log_2 5 = \frac{1}{\log_5 2} = 2$

(۲)

-۶

$\log_{15} 6 = \frac{\log_3 6}{\log_3 15} = \frac{\log_3 3 + \log_3 2}{\log_3 5 + \log_3 3} = \frac{1 + \frac{10}{16}}{1 + \frac{15}{10}} = \frac{\frac{26}{16}}{\frac{25}{10}} = \frac{13}{20}$

$\log_3 2 = \frac{1}{\log_2 3} = \frac{10}{16}$

(۲)

۷-

$$\log_{۸}^{۱۸} = \frac{\log_{۴}^{۱۸}}{\log_{۴}^{۸}} = \frac{\log_{۴}^{۲}+\log_{۴}^{۹}}{\frac{۳}{۲}\log_{۲}^{۲}} = m \Rightarrow ۲\log_{۴}^{۳} + \log_{۴}^{۲} = \frac{۳}{۲}m \to \log_{۴}^{۳} = \frac{۳m-۱}{۴}$$

۲

$$\log_{۴}^{۱۲} = \log_{۴}^{۳} + \log_{۴}^{۴} = \frac{۳m-۱}{۴} + ۱ = \frac{۳(m+۱)}{۴}$$

۸-

$$\left(\frac{۲}{۵}\right)^{۲x-۱} = \left(\frac{۵}{۲}\right)^{۳x^۲} \longrightarrow \left(\frac{۵}{۲}\right)^{۱-۲x} = \left(\frac{۵}{۲}\right)^{۳x^۲} \Rightarrow ۳x^۲+۲x-۱=۰ \begin{cases} x=-۱ \text{ غ ق ق} \\ x=\frac{۱}{۳} \end{cases}$$

$$\frac{۱۲۵}{۸} = \left(\frac{۵}{۲}\right)^{۳}$$

۲

$$\log_{۸}^{(۹\times\frac{۱}{۳}+۱)} = \frac{۲}{۳}\log_{۲}^{۲} = \boxed{\frac{۲}{۳}}$$

۹-

$$\log_{۸}^{b} = \frac{۲}{۳}\big(\underbrace{\log_{۲}^{۲}+\log_{۲}^{۳}}_{\log_{۲}^{۶}}\big) \Rightarrow b = (۲^{۳})^{\frac{۲}{۳}\log_{۲}^{۶}} = ۲^{۲\log_{۲}^{۶}} \xrightarrow{\text{change}} ۶^{۲\log_{۲}^{۲}} = ۳۶$$

۱

$$\log(۳\times۳۶-۸) = \log ۱۰۰ = \boxed{۲}$$

۱۰-

$$۲a = b+c \longrightarrow \frac{c}{a} = ۲-\frac{b}{a} \Rightarrow \frac{c}{a} = ۲-\frac{۴}{\log_{۱۰}۴} = \frac{۲\log۴-۴}{\log_{۱۰}۴}$$

$$\frac{-b}{-۴a} \xrightarrow{\text{معکوس}} \frac{۴a}{b} = \log_{۱۰}۴ \longrightarrow \frac{b}{a} = \frac{۴}{\log۴}$$

۲

$$(۲)^{-\frac{۱}{۸}\times\frac{۲\log۴-۴}{\log_{۱۰}۴}} = (۲)^{\frac{۲-\log۴}{۴\log۴}} \xrightarrow{۲=\log_{۱۰}^{۱۰۰}} ۲^{\frac{\log_{۱۰}^{\frac{۱۰۰}{۴}=۲۵}}{۴\log_{۱۰}۴}} = \left(۲^{\frac{۱}{۴}}\right)^{\log_{۴}^{۲۵}} \longrightarrow \left(۲^{\frac{۱}{۴}}\right)^{\log_{۲}^{۵}}$$

$$\Rightarrow \left(۲^{\log_{۲}^{۵}}\right)^{\frac{۱}{۴}} \xrightarrow{\text{change}} \left(۵^{\log_{۲}^{۲}}\right)^{\frac{۱}{۴}} = ۵^{\frac{۱}{۴}} = \sqrt[۴]{۵}$$