هستی رمضانی / یازدهم دختر C

①

$$\log_n^m = a \quad , \quad \log_{mn}^{m^2 n} = b \longrightarrow [b] = ?$$

$a > 0$

$$\log_n^m = a \longrightarrow m = n^a$$

$$\log_{n^a \times n} (n^a)^2 \times n = \log_{n^a \times n} n^a \times n \times n^a = \log_{n^a \times n} n^a \times n + \log_{n^a \times n} n^a = 1 + \log_{n^a \times n} n^a = b$$

$$[b] = [1 + \boxed{\log_{n^a \times n} n^a}] = [1, \dots] = 1$$

$a > 0 \to \log_{n^a \times n} n^a$: بین ۰ و ۱ است.

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②

الف) $y = \sqrt{\dfrac{x}{\log_{\frac{1}{2}} x}}$

$\log_{\frac{1}{2}} x \longrightarrow x > 0$

$\dfrac{x}{\log_{\frac{1}{2}} x} \geq 0 \to x = 0$, $x = 1$

x		0		1	
	ت ن	0	+	ت ن	−

جواب

$$D = [0, 1)$$

ب) $y = \dfrac{\log_2 (x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$

$x^2 - x - 2 > 0 \longrightarrow (x - 2)(x + 1) > 0$ ، ریشه‌ها: 2 و -1

$\sqrt{x^2 - 1} + 1 \neq 0$

$\sqrt{x^2 - 1} \neq -1$ همیشه برقرار است.

x		−1		2	
	+	0	−	0	+

$$\Longrightarrow D = (-\infty, -1) \cup (2, +\infty)$$

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③

$$2 \log_x^a + \log_a^{\sqrt{x}} = 2 \qquad x = 9 \qquad a = ?$$

$$2 \log_9 a + \log_a 3 = 2 \longrightarrow \frac{2}{2} \log_3 a + \boxed{\log_a 3} = 2 \longrightarrow \frac{1}{t} + t = 2$$

$\log_a 3 \to t$

$$t^2 - 2t + 1 = 0 \longrightarrow (t - 1)^2 = 0 \longrightarrow t = 1 \longrightarrow \log_a 3 = 1 \longrightarrow a^1 = 3 \Longrightarrow \boxed{a = 3}$$

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$$\left(\log \tfrac{5}{3}\right)x^2 + (\log 6)x - \log 15 = 0$$

$$(\log 5 - \log 3)x^2 + (2\log 3)x - (\log 5 + \log 3) = 0$$

$$\log 5 = 1 - \log 2 \longrightarrow \log 5 = 0.7$$

$$0.3x^2 + 0.8x - 1.1 = 0 \longrightarrow 3x^2 + 8x - 11 = 0 \qquad |\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{64 + 132}}{3}$$

$$\Longrightarrow \boxed{|\alpha - \beta| = \frac{14}{3}}$$

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$$\log_{14} 10 = \frac{\log_2 10}{\log_2 14} = \frac{\log_2 5\times 2}{\log_2 7\times 2} = \frac{\log_2 5 + 1}{\log_2 7 + 1} = \frac{\frac{10}{5} + 1}{2.8 + 1} = \boxed{\frac{3}{3.8}}$$

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$$\log_{15} 6 = \frac{\log_2 6}{\log_2 15} = \frac{\log_2 3 + 1}{\log_2 5 + \log_2 3} = \frac{1.6 + 1}{1.6 \times 2.5} = \frac{2.6}{4} = \boxed{\frac{13}{20}}$$

$$\log_3 5 \times \log_2 3 = \log_2 5 = 1.6 \times 1.5$$

7

$$\log_8 18 = m \qquad \log_4 12 = ?$$

$$\log_{2^3} 2\times 3^2 = \log_{2^3} 2 + \log_{2^3} 3^2 = \frac{1}{3} + \frac{2}{3}\log_2 3 = m \longrightarrow \frac{2}{\cancel{3}}\log_2 3 = \frac{3m-1}{\cancel{3}} \longrightarrow \log_2 3 = \frac{3m-1}{2}$$

$$\log_{2^2} 2^2\times 3 = \frac{1}{2}\log_2 3 + 1 \longrightarrow \frac{1}{2}\times\frac{3m-1}{2} + 1 \longrightarrow \frac{3m-1+4}{4} = \frac{3m+3}{4} = \boxed{\frac{3(m+1)}{4}}$$

8

$$(0.4)^{2x-1} = \left(\frac{125}{8}\right)^{x^2} \longrightarrow \left(\frac{2}{5}\right)^{2x-1} = \left(\frac{5^3}{2^3}\right)^{x^2} \longrightarrow \left(\frac{5}{2}\right)^{1-2x} = \left(\frac{5}{2}\right)^{3x^2}$$

$$3x^2 = 1 - 2x \longrightarrow 3x^2 + 2x - 1 = 0 \longrightarrow x = -1,\ x = \frac{1}{3}$$

$$\log_8(9x+1) \xrightarrow{x=-1} \log_8(-8) = \times$$

$$\log_8(9x+1) \xrightarrow{x=\frac{1}{3}} \log_8 4 = \boxed{\frac{2}{3}}$$

9

$$\log_8^b = \frac{2}{3}(1+a) \qquad \log_2^3 = a \longrightarrow 3 = 2^a$$

$$b = 4^{(1+a)} = 2^{2+2a} = 4 \times \boxed{2^{2a}} = 36$$

$$\boxed{2^{2a}} \to 9$$

$$\log(3 \times 36 - 8) = \log 100 = \boxed{2}$$

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$$\frac{1}{a+b} = \log 4 \longrightarrow \frac{4a}{b} = \log 4 \qquad 2a = b+c \xrightarrow{\div a} 2 = \frac{b}{a} + \frac{c}{a}$$

$$\longrightarrow \frac{b}{a} = 4\log_4^{10}$$

$$2 - 4\log_4^{10} = \frac{c}{a} \longrightarrow -2\log_2^5 = \frac{c}{a} \qquad \left(2^{-\frac{1}{8}}\right)^{-\log_2^5} = 2^{\log_2^{\sqrt[8]{25}}} = \sqrt[8]{25}$$

$$= \boxed{\sqrt[4]{5}}$$