(۱) $b = \log_{mn}^{m^2 n} = \log_{n}^{n^{2a+1}} = \frac{2a+1}{a+1} \to [b] = \left[\frac{2a+1}{a+1}\right] = 1$

$\left[2 - \frac{1}{a+1}\right]$

$0 < \frac{1}{a+1} < 1$

۱/۵

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(۲) الف)

$\frac{x}{\log_{\frac{1}{2}}^{x}} \geq 0 \to \log_{2}^{x} < 0 \to x < 2^0 = 1 \to \begin{cases} x > 0 \\ x < 1 \end{cases} \to D = (0,1)$

۱

ب)

$\sqrt{x^2 - 1} \to (-\infty, -1] \cup [1, +\infty)$ ①

$\log_{4}^{x^2 - x - 2} \to x^2 - x - 2 > 0 \to (-\infty, -1) \cup (2, +\infty)$ ②

$\xrightarrow{①\cap②} (-\infty, 1) \cup (2, +\infty)$

ناقص

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(۳) $\xrightarrow{x=9} 2\log_{9}^{a} + \log_{a}^{3} = 2 \to \log_{3}^{a} + \log_{a}^{3} = 2$

$\log_{3}^{a} = 1 \quad a = 3 \quad t + \frac{1}{t} = 2$

۱/۵

$\log_{a}^{a} = A \qquad \frac{2}{A} + \frac{1}{2}A = 2 \to A^2 - 4A + 4 = 0 \to A = 2$

$t = 1$

$2 = \log_{a}^{9} \to 9 = a^2 \to a \begin{cases} a = 3 \checkmark \\ a = -3 \times \end{cases}$

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(۴) ب)

$\log^{5} = 0/7 \quad \log^{9} = 0/8 \quad \log^{15} = 1/1 \quad \log^{\frac{5}{3}} = 0/3$

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$\to 0/3x^2 + 0/8x - 1/1 \to \begin{cases} x_1 = 1 \\ x_2 = \frac{-1/1}{0/3} \end{cases} \to x_1 - x_2 = \frac{14}{3}$

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(۵) $\log_{14}^{10} = \log_{7 \times 2}^{2 \times 5} = \frac{3}{\frac{19}{5}} = \frac{15}{19}$

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(۶)

$$\frac{\log_3 6}{\log_3 15} = \frac{\log_3 2 + \log_3 3}{\log_3 5 + \log_3 3} = \frac{\frac{1}{1.6}+1}{1.5+1} = \frac{13}{20} = 0.65$$

(۵)

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(۷) $\log_4 12 = \log_{2^2} 2\times 3 \xrightarrow{2} 2^{\frac{3m-1}{2}} \quad \log_2 2^{\frac{3m+3}{2}} = \frac{3m+3}{4}$

$8^m = 18 \to 2^{3m} = 2\times 3^2 \to 2^{3m-1} = 3^2 \to 2^{\frac{3m-1}{2}} = 3$

(۵)

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(۸) $\left(\frac{5}{2}\right)^{-2x+1} = \left(\frac{5}{2}\right)^{3x^2} \to 3x^2+2x-1=0 \quad x = \begin{cases} -1 \times \\ \frac{1}{3} \checkmark \end{cases}$

$\to \log_8 2x+1 = \frac{2}{3}$

(۵)

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(۹) $b = 8^{\frac{2}{3}(1+a)} = 2^{2(1+a)} = 2^{2+2a} = 2^2 \times (2^a)^2$

$b = 4\times 3^2 = 36$

$\log_{10}(3b-8) = \log_{10} 100 = 2$

(۵)

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(۱۰) $\frac{1}{\alpha+\beta} = \log 5 \to \frac{4a}{b} = 2\log 2 \to \frac{a}{b} = \frac{\log 2}{2}$

$a = \frac{b+c}{2} \to 2a = b+c \xrightarrow{\div a} 2 = \frac{b}{a} + \frac{c}{a} \to \frac{c}{a} = 2 - \frac{b}{a} = 2 - \frac{2}{\log 2}$

$\left(2^{-\frac{1}{8}}\right)^{\frac{c}{a}} = 2^{-\frac{1}{4}+\frac{1}{4\log 2}} = 2^{-\frac{1}{4}} \times 2^{\frac{\log 2}{4}} = 2^{-\frac{1}{4}} \times 10^{\frac{1}{4}} = \frac{10^{\frac{1}{4}}}{2^{\frac{1}{4}}} = 5^{\frac{1}{4}} = \sqrt[4]{5}$

(۵)