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نیایش شاه‌نظری　　　　یازدهم ریاضی e　　　　تکلیف کد ۲۳

① $\log_{mn} m^2n = b = \log_{mn} mn + \log_{mn} m = 1 + \dfrac{1}{\log_m mn} = 1 + \dfrac{1}{\log_m m + \log_m n}$

$1 + \dfrac{1}{1 + \dfrac{1}{\log_n m}} = 1 + \dfrac{a}{a+1} = b \rightarrow [b] = \left[1 + \dfrac{a}{a+1}\right] = \left[\dfrac{a}{a+1}\right] + 1 = 1$

$0 < \dfrac{a}{a+1} < 1$

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② الف) $y = \sqrt{\dfrac{x}{\log_{\frac{1}{2}} x}}$

① $x > 0$

② $\dfrac{x}{\log_{\frac{1}{2}} x} \geq 0$　　$-\ \overset{0}{|}\ +\ \overset{1}{|}\ -$　$\rightarrow x \in [0, 1)$

$① \cap ② \rightarrow 0 < x < 1 \rightarrow \boxed{D_f = (0, 1)}$

ب) $y = \dfrac{\log_2 (x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$

① $x^2 - x - 2 > 0$
$(x-2)(x+1)$　　$+\ \overset{-1}{|}\ -\ \overset{2}{|}\ +$

$x \in (-\infty, -1) \cup (2, +\infty)$

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② $x^2 - 1 \geq 0 \rightarrow x^2 \geq 1 \rightarrow x \in (-\infty, -1] \cup [1, +\infty)$

$① \cap ② \rightarrow x \in (-\infty, -1) \cup (2, +\infty) \rightarrow \boxed{D_f = \mathbb{R} - [-1, 2]}$

(۳)

$$\log_x a + \log_a \sqrt{x} = 2 \qquad x = 9 \rightarrow$$

$$2\log_9 a + \log_a 3 = 2$$

$$\underbrace{\log_3 a}_{m} + \underbrace{\log_a 3}_{\frac{1}{m}} = 2 \rightarrow m + \frac{1}{m} - 2 = 0$$

$$m^2 - 2m + 1 = 0 \rightarrow m = 1$$

$$m = \log_3 a = 1 \rightarrow a = 3$$

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(۴)

$$(\log \tfrac{5}{3})x^2 + (\log 9)x - \log 15 = 0$$

$$|x' - x''| = \frac{\sqrt{\Delta}}{|a|} = \frac{(\log 9)^2 - 4(-\log 15)(\log \frac{5}{3})}{\log \frac{5}{3}}$$

$$\frac{(2\log 3)^2 + 4\left(\overset{0.4}{\log 3} + (\overset{0.7}{\log 10 - \log 2})\right)\left[(\overset{0.7}{\log 10 - \log 2}) - \overset{0.4}{\log 3}\right]}{\log 10 - \log 2 - \log 3} = \frac{\sqrt{0.64 + 4(1.1)(0.3)}}{0.3} =$$

$$\frac{\sqrt{1.96}}{0.30} = \frac{1.4}{0.3} = \frac{14}{3}$$

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(۵)

$$\log_{14} 10 = \frac{\log_2 10}{\log_2 14} = \frac{\log_2 2 + \log_2 5}{\log_2 2 + \log_2 7} = \frac{1 + \frac{1}{0.5}}{1 + 2.8} = \frac{3}{3.8} = \frac{15}{19}$$

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(۶)

$$\log_{15} 6 = \frac{\log_3 6}{\log_3 15} = \frac{1 + \log_3 2}{1 + \log_3 5} = \frac{1 + \frac{1}{1.6}}{2.5} = \frac{13}{20} = 0.65$$

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(۷)

$$\log_4 12 = \frac{\log_2 12}{\log_2 4} = \frac{2 + \log_2 3}{2} = \frac{2 + \frac{3m-1}{2}}{2} = \frac{3m+3}{4}$$

$\log_8 18 = m \rightarrow \log_2 18 = 3m \rightarrow 1+2\log_2 3 = 3m \rightarrow \log_2 3 = \frac{3m-1}{2}$

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(۸)

$\left(\frac{2}{5}\right)^{2x-1} = \left(\frac{125}{8}\right)^{x^2} \rightarrow \left(\frac{5}{2}\right)^{1-2x} = \left(\frac{5}{2}\right)^{3x^2}$

$3x^2 = 1-2x \rightarrow 3x^2+2x-1=0 \rightarrow x=-1 \quad , \quad x=\frac{1}{3}$ ۲)

$x=-1 \rightarrow \log_8 {-8} \rightarrow$ غ ق ق

$x=\frac{1}{3} \rightarrow \log_8 4 = \log_{2^3} 2^2 = \frac{2}{3}$

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(۹)

$\log_2 3 = a \qquad \log_8 b = \frac{2}{3}(1+a) = \frac{2}{3}(\log_2 6) = \log_8 36$ ۲)

$b=36 \rightarrow \log 3b - 8 = \log 100 = 2$

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(۱۰)

$-4ax^2+bx+\frac{1}{4}c=0 \qquad \frac{1}{x_1+x_2} = \log 4 \rightarrow \frac{b}{4a} = \log_4 10 = \frac{1}{2}\log_2 10$

$b = 2a\log_2 10$ ۲)

$a = \frac{b+c}{2} \rightarrow 2a = 2a\log_2 10 + c \rightarrow c = 2a(1-\log_2 10)$

$c = 2a(\log_2 0.2)$

$\left(\frac{1}{\sqrt[8]{2}}\right)^{\frac{c}{a}} = \left(\frac{1}{2}\right)^{\frac{1}{8}\left(\frac{c}{a}\right)} = \frac{1}{2}^{\frac{1}{8}\left(\frac{2a\log_2 0.2}{a}\right)} = \left(\frac{1}{2}\right)^{\frac{1}{4}\log_2 0.2} = 2^{-\frac{1}{4}\log_2 0.2}$

$\boxed{\sqrt[4]{5}} \leftarrow 5^{\frac{1}{4}} \leftarrow \left(\frac{10}{2}\right)^{\frac{1}{4}} \qquad \Leftarrow 0.2^{-\frac{1}{4}} = 2^{-\frac{1}{4}\log_2 0.2}$