بنام خداوند دانا

پرنیا بابندیان

۱۹,۷۵

①

$$f(x) = 3^{Ax+B} \qquad y = x^2 \qquad (1,1) \quad (3,9)$$

نقطه تلاقی با محور y

$(0,y)$

$$(1,1) \to (1)^2 = 3^{A+B} \Rightarrow 3^0 \Rightarrow A+B=0$$

$$(3,9) \to (3)^2 = 3^{3A+B} \Rightarrow 3A+B=2$$

$$2A=2 \Rightarrow A=1$$

$$\Rightarrow B=-1$$

$$f(x) = 3^{x-1} \xRightarrow{x=0} 3^{-1} = \frac{1}{3}$$

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②

$$\log_2 (4^x+15) = x+3$$

$$\underbrace{2^{x+3}}_{8\times 2^x} = 4^x + 15$$

$$2^{2x} - 8\times 2^x + 15 = 0$$

$$(2^x-3)(2^x-5) = 0 \Rightarrow 2^x = 3 \Rightarrow \log_2 2^x = \log_2 3 \Rightarrow \boxed{x = \log_2 3}$$

$$\Rightarrow 2^x = 5 \Rightarrow \log_2 2^x = \log_2 5 \Rightarrow \boxed{x = \log_2 5}$$

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$\log_2 3 + \log_2 5 = \log_2 15$

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③

$$(\log_{21} 3)^2 + \log_{21} (147) + \log_{21} 1323 = 4 + (\log_{21} 3)^2 = 4 + \frac{1}{(1+\log_3 7)^2}$$

$$\log_{21} 3\times 7^2 = \log_{21} 3 + 2\log_{21} 7$$

$$\log_{21} 1323 = \log_{21} 3^3\times 7^2 = 3\log_{21} 3 + 2\log_{21} 7$$

$$\Rightarrow 4(\underbrace{\log_{21} 3 + \log_{21} 7}_{\log_{21} 21}) = 4$$

$$\log_{21} 3 = \frac{1}{\log_3 3 + \log_3 7} = \frac{1}{1+\log_3 7}$$

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④

$$\log (\overbrace{x^2-2x+1}^{(x-1)^2}) + 3\log (1-x)$$

$$\log_3 9 = 2$$

$$= 2\log (x-1) + 3\log (1-x) = \log (x-1)^2 + \log \overbrace{(1-x)^3}^{-(x-1)^3}$$

$$= \log -(x-1)^5 = 5 \Rightarrow \log (1-x)^5 = 5 \Rightarrow 5\log (1-x) = 5 \Rightarrow \log_{10} (1-x) = 1$$

$$1-x = 10 \Rightarrow \boxed{x = -9}$$

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به نام خداوند دانش

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$$\log_2^{x^2+2x+4} + \log_2^{(x-2)} = 3$$

$$\Delta = 4 - 4 \times 1 \times 4 < 0$$

ریشه ندارد

$$\log_2^{(x^2+2x+4)(x-2)} = 3 \Rightarrow (x^2+2x+4)(x-2) = 8$$

$$x^3 - 2x^2 + 2x^2 - \cancel{4x} + \cancel{4x} - 8 = 8$$

$$x^3 = 16 \Rightarrow x = \sqrt[3]{2^4} = 2\sqrt[3]{2}$$

$$\log_{\sqrt[3]{2}} \sqrt[3]{2^4} = 4$$

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(۶)

$$\log(2-x) - \log\frac{1}{(x-2)^2} = 3$$

$$(x-2)^2 = (2-x)^2$$

$$\log(2-x) - \log(2-x)^{-2} = \log(2-x) + 2\log(2-x) = 3$$

$$3\log(2-x) = 3 \Rightarrow \log(2-x) = 1$$

$$2-x = 10 \Rightarrow \boxed{x = -8}$$

$$\log_{\sqrt{2}} 8 = 6$$

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(۷)

$$3^{x^2-2} = (3^4)^x \Rightarrow x^2-2 = 4x$$

$$x^2-4x-2 = 0 \quad +6 \quad +6$$

$$x^2-4x+4 = 6$$

$$(x-2)^2 = 6 \Rightarrow |x-2| = \sqrt{6} \Rightarrow x = 2$$

$$\Rightarrow 2 \pm \sqrt{6}$$

$$- \;\; \text{غ ق ق}$$

$$\Delta = 16 - 4\times1\times-2 = 24$$

$$\sqrt{\Delta} = 2\sqrt{6}$$

$$\frac{4 \pm 2\sqrt{6}}{2} = 2 \pm \sqrt{6}$$

$$- \;\; \text{غ ق ق}$$

$$\log_6^{2+\sqrt{6}-2} = \frac{1}{2}$$

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به نام خداوند هستی

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$\log_3^2 = \frac{5}{8} \Rightarrow \log_2^3 = \frac{8}{5}$

$\log_{18}^8 = \log_{18}^{2^3} = 3\log_{18}^2 = \frac{3}{\log_2^{18}} = \frac{3}{\log_2^{3^2\times 2}} = \frac{3}{2\log_2^3 + \cancel{\log_2^2}_{1}}$ ۱

$\frac{3}{\frac{16}{5}+1} = \frac{3}{\frac{21}{5}} = \frac{15}{21} = \boxed{\frac{5}{7}}$

$= \frac{1}{2}\log_2^3 = 0.8 \Rightarrow \log_2^3 = 1.6 \Rightarrow \log_3^2 = \frac{1}{1.6} = \frac{10}{16} = \frac{5}{8}$

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$\log_4^3 = 0.8 \qquad \log_{12}^6 = \log_{12}^2 + \log_{12}^3 = \frac{5}{18} + \frac{4}{9} = \boxed{\frac{13}{18}}$ ۱

$\log_{12}^2 = \frac{1}{\log_2^{12}} = \frac{1}{2\cancel{\log_2^2}_{1} + \log_2^3} = \frac{1}{2+\log_2^3} = \frac{1}{3.6} = \frac{1}{\frac{36}{10}} = \frac{10}{36} = \frac{5}{18}$

$\log_{12}^3 = \frac{1}{\log_3^{12}} = \frac{1}{2\log_3^2 + \cancel{\log_3^3}_{1}} = \frac{1}{\frac{5}{4}+1} = \frac{1}{\frac{9}{4}} = \frac{4}{9}$

۱۰

$(a\log 2)x^2 + ax + b\log 2 = 0 \qquad -1 \qquad (\sqrt{2})^{\frac{b}{a}}$

$a\log 2 - a + b\log 2 = 0$

$a(\log 2 - 1) + b\log 2 = 0 \xRightarrow{\div a} \log 2 - 1 + \frac{b}{a}\log 2 = 0$ ۱

$\frac{\log 2 - 1}{\log 2} + \frac{b}{a} = 0 \Rightarrow \frac{b}{a} = \frac{\log 2 - 1}{-\log 2} = \frac{1-\log 2}{\log 2}$

$= \frac{\log_{10}^{10} - \log 2}{\log 2}$

$= \frac{\log_{10}^5}{\log_{10}^2} = \log_2^5$

$\Rightarrow \qquad (\sqrt{2})^{\frac{\log 2 + 1}{-\log 2}} \qquad 2^{\frac{\log_2^5}{2}}$

$= 2^{\frac{\log_2^5}{\log_2^4}} = 2^{\log_4^5} \quad \frac{1}{2}\log_2^5 \quad = 5 \quad = \sqrt{5}$