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زرانو صدرالعلمایی — یازدهم دختر — تکلیف ۲۴

۱-

$f(x) = 3^{Ax+B}$ $\quad y = x^2$

$A\begin{vmatrix}1\\1\end{vmatrix} \quad B\begin{vmatrix}3\\9\end{vmatrix} \longrightarrow \begin{cases}1 = 3^{A+B} = 3^0 \rightarrow A+B=0\\ 9 = 3^{3A+B} \quad\quad 3A+B=2\end{cases}$

$2A = 2 \rightarrow A=1 \quad B=-1$

$\longrightarrow f(x) = 3^{x-1} \xrightarrow{x=0} 3^{-1} = \boxed{\frac{1}{3}}$ نقطه برخورد با محور y

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$\log_2^{(4^x+15)} = x+3$ $\quad 2^{x+3} = 2^{2x} + 15 \longrightarrow 2^{2x} - 2^3(2^x) + 15 = 0$

$t^2 - 8t + 15 = 0 \quad (t-3)(t-5) = 0$

$\longrightarrow 2^x = 3 \quad , \quad 2^x = 5$

$\log_2^3 = x \quad\quad \log_2^5 = x \longrightarrow \log_2^5 + \log_2^3 = \boxed{\log_2^{15}}$

۳-

$(\log_{21}^3)^2 + \log_{21}^{147} \log_{21}^{1323}$

$(2\log_{21}^7 + \log_{21}^3)(3\log_{21}^3 + 2\log_{21}^7)$ $\quad \log_{21}^{21} - \log_{21}^3 = \log_{21}^7$, $1 - t$

$t^2 + (2-2t+t)(3t+2-2t) \rightarrow t^2 + (-t+2)(t+2)$

$t^2 + (4-t^2) = \boxed{4}$

۴-

$\log^{(x^2-2x+1)} + 3\log^{(1-x)} = 5\log_{10}^{10}$ $\quad (x^2-2x+1)(1-x)^3 = 10^5$

$(x-1)^2 = (1-x)^2(1-x)^3 = 10^5 = (1-x)^5 = 10^5$

$1 - x = 10 \rightarrow x = -9$

$\log_3^{(-x)} = \log_3^{3^2} = ②$

۵-

$\log_2^{(x^2+2x+4)} + \log_2^{x-2} = ③ = \log_2^{2^3}$ $\quad (x^2+2x+4)(x-2) = 8$

$x^3 - 8 = 8 \rightarrow x^3 = 16 \rightarrow x = \sqrt[3]{16}$

$\log_{\sqrt{2}}^{\sqrt[3]{16}} \quad \log_2^{16} = ④$

۶-

$\log^{(2-x)} - \log^{\frac{1}{(x-2)^2}} = 3 \longrightarrow \log^{\frac{(2-x)}{\frac{1}{(x-2)^2}}} = \log^{(2-x)^3} = \log_{10}^{10^3} \longrightarrow 2-x = 10 \rightarrow x = -8$

$\log_{\sqrt{2}}^{(-x)} \rightarrow \log_{2^{\frac{1}{2}}}^{2^3} \rightarrow ⑥$

۷-

$3^{x^2-2} = 81^x = 3^{4x}$ $\qquad \log_6^{x-2} \longrightarrow \log_6^{2+\sqrt6-2} = \log_6^{\sqrt6} = \frac{1}{2}$

$x^2-4x-2=0$

(۲)

$x = \frac{4\pm\sqrt{16+8}}{2} = \frac{4\pm2\sqrt6}{2} = 2\pm\sqrt6$

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$\log_3^2 = \frac{5}{8} \qquad \log_{18}^8 \longrightarrow \frac{3\times\log_3^2}{\log_3^{18}} = \frac{3\times\frac58}{\log_3^{3^2}+\log_3^2} = \frac{3\times\frac58}{2+\frac58} = \frac{\frac{15}{8}}{\frac{21}{8}} = \frac{15}{21} = \frac{5}{7}$ (۲)

۹-

$\log_4^3 = 0.8 \qquad \log_{12}^6 = ? = \frac{\log_4^6}{\log_4^{12}} = \frac{\log_4^2+\log_4^3}{\log_4^3+\log_4^4} = \frac{\frac12 + 0.8}{0.8+1} = \frac{1.3}{1.8} = \frac{13}{18}$ (۲)

۱۰-

$(a\log^2)x^2 + ax + b\log^2 = 0 \qquad x=-1 \qquad (\sqrt2)^{\frac ba} = 2^{\frac12\cdot\frac ba} = 5^{\frac12} = \sqrt5$

(۲)

$a\log^2 - a + b\log^2 = 0 \longrightarrow \log^{2^a} + \log^{2^b} - a = 0 = \log^{2^a\times2^b} = a \qquad 2^{a+b} = 10^a$

$2^{b\times\frac1a} = 5^{a\times\frac1a} = 2^{\frac ba} = 5$