س۱

$A+B=0$

$3A+B=2$

$2A=2 \rightarrow A=1 \;,\; A+B=0 \xrightarrow{A=1} B=-1$

نقاط تقاطع $\rightarrow \begin{vmatrix} 3 & 1 \\ 9 & 1 \end{vmatrix}$ و

عرض نقطه تلاقی تابع با محور yها $= \frac{1}{3}$

س۲

$4^x+10\times 2^x\times 8 \rightarrow (2^x-3)(2^x-5)=0$

$x_1=\log_2^3$ ، $x_2=\log_2^5$

$\Rightarrow \log_2^3+\log_2^5=\log_2^{15}$

س۳

$\left(\log_{21}^3\right)+\log_{21}^{(17)}\log^{(13\times 3)}=0 \Rightarrow (\log_{21}^3)^2+4-(\log_{21}^3)^2=0$

$(1+\log_{21}^7)(2+\log_{21}^3)=(2-\log_{21}^3)(2+\log_{21}^3)$

$1-\log_{21}^3$

$(\log_{21}^3)^2-6=0$

س۴

$\log(x^2-2x+1)+3\log(1-x)=5$

$\log^{(x-1)^2(1-x)^3}=5 \rightarrow \log^{(1-x)^5}=5 \rightarrow 10^5=(10x)^5 \rightarrow 10=1-x$

$x=-9$

$\rightarrow x=-9 \Leftrightarrow \log_3^{(-9)}=2$

س۵

$\log_2^{(x^2+2x+6)}+\log_2^{(x-2)}=3$

$\log^{x^3-8}=3 \rightarrow x^3-8=8 \rightarrow x=2\sqrt[3]{2} \Rightarrow \log_{3\sqrt[3]{2}}^{2\sqrt[3]{2}}=3$

(ابتدا مخرج را منفی کنید)

$\log^{(2-n)}_{(n-2)^2} = 3$ $\quad$ $\log^{(-n)}_{\sqrt{2}} = 6 \Rightarrow \log^{-(-n)}_{\sqrt{2}} = 6$

س ۶:

$\log^{(2-n)}_{\frac{1}{(n-2)^2}} = \log^{(2-n)}_{(n-2)^{-2}} = -2\log^{(2-n)}_{(2-n)}$

$\Rightarrow \log^{2-n}_{2-n} \longrightarrow 2-n = 1 \rightarrow n = -1$

س ۷:

$3^{n^2-2n} = 81 = 3^4 \Rightarrow n^2-2n = 4 \rightarrow n^2-2n-4 = 0 \rightarrow (n-2)^2 = 6 \rightarrow n-2 = \sqrt{6}$

$\log^{(n-2)}_{2} = 0 \Rightarrow \log^{\sqrt{2}}_{4} = \frac{1}{2}$

س ۸:

$\log^2_{81} = \frac{5}{8} \rightarrow \frac{\log^2_3}{\log^2_{81}} = \frac{5}{8} \rightarrow \log^2_3 = 5a$

$\log^3_2 = 8a$

$\log^{81}_{12} = ? \rightarrow \frac{3\log^2_3}{\log^2_{12}} = \frac{3(5a)}{\log^2_{4\times3}} = \frac{3(5a)}{2\log^2_2 + \log^2_3} = \frac{3(5a)}{2(8a)+5a}$

س ۹:

$\log^{\sqrt{3}}_{\frac{1}{3}} = \log^{\sqrt{3}}_{3^{-1}} = -2\log^{\sqrt{3}}_{3} = -4$

$\log^{\sqrt{3}}_{\frac{1}{3}} = \frac{\log^2_{\frac{1}{3}}}{\log^2_{\sqrt{3}}} = \frac{-\log^2_3}{\frac{1}{2}\log^2_3} = -2$

س ۱۰:

$\log^{12}_{4} = ?$

$\log^3_{10} = 0.4771 \rightarrow \log^3_{10} = 4771$

$\log^2_3 = 3010$

$\log^2_{12} = \log^2_{4\times3} = 2\log^2_2 + \log^2_3 = 2(3010) + 4771 = 10791$

$\log^{12}_4 = \frac{\log^2_{12}}{\log^2_4} = \frac{10791}{6020} \approx 1.792$

س ۱۱:

$\log^2_{10} = a \rightarrow \log^2_5 = 1 - a$

$\log^{10}_{20} = \frac{\log^2_{10}}{\log^2_{20}} = \frac{\log^2_{10}}{\log^2_{2\times10}} = \frac{\log^2_{10}}{1 + \log^2_{10}} = \frac{a}{1+a}$

$\log^{20}_{50} = \frac{\log^2_{50}}{\log^2_{20}} = \frac{\log^2_{5\times10}}{\log^2_{2\times10}} = \frac{1 + \log^2_5}{1 + \log^2_{10}} = \frac{2-a}{1+a}$

$\frac{\log^2_3 + \log^2_{10}}{2\log^2_2 + \log^2_3} = \frac{30a + 18}{10a + 18a} = \frac{48a}{28a} = \frac{12}{7}$

س ۱۲:

$(\log^2_3)x^2 + ax + b\log^2_3 = 0$

$x = -1 \rightarrow a\log^2_3 - a + b\log^2_3 = 0 \rightarrow (a+b)\log^2_3 = a \rightarrow \log^2_3 = \frac{a}{a+b}$

$\rightarrow \frac{1}{\log^{10}_2} = \frac{a}{a+b} \rightarrow \log^{10}_2 = \frac{a+b}{a} = 1 + \frac{b}{a} \rightarrow \log^{10}_2 - 1 = \frac{b}{a}$

$\rightarrow \frac{b}{a} = \log^2_{\sqrt{5}} \Rightarrow (\sqrt{2})^{\log^{\sqrt{5}}_2} = 2^{\frac{1}{2}\log^{\sqrt{5}}_2} = \sqrt{5}$