زهرا حبیبی / تکلیف ۲۴ / یازدهم تجربی

۲۰

۱)

$y = 3^{Ax+B} \xrightarrow{(1,1)} 1 = 3^{A+B} \to A+B=0$

$\xrightarrow{(3,9)} 9 = 3^{3A+B} \to 3A+B=2$

$\to \boxed{A=1}, \boxed{B=-1}$

$\xrightarrow{(0,y)} y = 3^{0-1} = \boxed{\frac{1}{3}}$

۲)

$\log_2^{4^x+15} = x+3 \to 2^{x+3} = 4^x + 15 \quad (4^x = 2^{2x}) \to 2^{x+3} - 2^{2x} = 15 \to 2^x(2^3 - 2^x) = 15$

$2^x = t$

$\to t(8-t) = 15 \to t^2 - 8t + 15 = 0 \to (t-3)(t-5) = 0 \to t=3, \ t=5$

$\to 2^x = 3 \to x = \log_2^3$

$\to 2^x = 5 \to x = \log_2^5$

$\log_2^3 + \log_2^5 = \boxed{\log_2^{15}}$

۳)

$(\log_{21}^3)^2 + \log_{21}^{147} \log_{21}^{1323} \to \log_{21}^{3^3 \times 7^2} = 2\log_{21}^7 + 3\log_{21}^3$

$\log_{21}^{7^2 \times 3} = 2\log_{21}^7 + \log_{21}^3 \to \log_{21}^{21} = \log_{21}^3 + \log_{21}^7 \xrightarrow{\log_{21}^3 = a} \log_{21}^7 = 1-a$

$\to a^2 + (2-2a+a)(2-2a+3a) = a^2 + \underbrace{(2-a)(a+2)}_{4-a^2} = a^2 - a^2 + 4 = \boxed{4}$

۴)

$\log^{x^2-2x+1} + 3\log(1-x) = 5 \to \log^{(x^2-2x+1)(1-x)^3} = 5$

$\Rightarrow \log_{10}(1-x)^5 = 5 \to 10^5 = (1-x)^5 \to x=-9 \to \log_3^{-x} = \log_3^9 = \boxed{2}$

۵)

$\log_2^{x^2+2x+4} + \log_2^{x-2} = 3 \Rightarrow \log_2^{(x-2)(x^2+2x+4)} = 3 \to \log_2^{x^3-8} = 3 \quad (8 = x^3 - 8)$

$\to x^3 = 16 \to x = \sqrt[3]{16} = 2^{\frac{4}{3}}$

$\to \log_{\sqrt[3]{2}}^{x} = \log_{2^{\frac{1}{3}}}^{2^{\frac{4}{3}}} = \boxed{4}$

(۶)

$\log(2-x) - \log \frac{1}{(x-2)^2} = 3 \quad \log \frac{2-x}{\frac{1}{(x-2)^2}} = 3 \longrightarrow (2-x)^3$

$10^3 = (2-x)^3 \longrightarrow \boxed{x=-8} \quad \log_{\sqrt{2}} 8 = \boxed{6}$

(۷)

$3^{x^2-2} = 3^{4x} \rightarrow x^2-2=4x \rightarrow x^2-4x-2=0 \rightarrow x = \frac{4 \pm \sqrt{24}}{2} = 2 \pm \sqrt{6}$

$\log_6^{x-2} = \log_6^{\sqrt{6}} = \boxed{\frac{1}{2}}$

(۸)

$\log_{18}^{8} = \frac{\log_3^8}{\log_3^{18}} = \frac{3\log_3^2}{2\log_3^3+\log_3^2} = \frac{\frac{15}{8}}{2+\frac{5}{8}} = \boxed{\frac{5}{7}}$

(۹)

$\log_{12}^{9} = \frac{\log_4^9}{\log_4^{12}} = \frac{\log_4^3 + \log_4^2}{\log_4^4+\log_4^3} = \frac{0.8+0.5}{1+0.8} = \boxed{\frac{13}{18}}$

(۱۰)

$(a\log 2)x^2 + ax + b\log 2 = 0 \qquad a+c=b \leftarrow$ است (-1) یکی از ریشه‌ها

$a\log 2 + b\log 2 = a \rightarrow a - a\log 2 = b\log 2$

$\rightarrow a(1-\log 2) = b\log 2 \Rightarrow \frac{b}{a} = \frac{1-\log 2}{\log 2} = \frac{\log 5}{\log 2}$

$\log_{10}^{10} - \log_{10}^{2} = \log_{10}^{5} \rightarrow \log_2^5$

$(\sqrt{2})^{\frac{b}{a}} = 2^{\frac{1}{2}\times\frac{b}{a}} = 2^{\frac{1}{2}\log_2^5} = 5^{\frac{1}{2}\log_2^2} = 5^{\frac{1}{2}} = \boxed{\sqrt{5}}$