زهرا مجیدی / تکلیف ۲۴ / یازدهم تجربی

(1)

$$y=3^{Ax+B}\xrightarrow[(3,9)]{(1,1)} 1=3^{A+B}\to A+B=0$$
$$9=3^{3A+B}\to 3A+B=2 \quad \Big\}\to \boxed{A=1},\ \boxed{B=-1}$$
$$\xrightarrow{(0,y)} y=3^{0-1}=\boxed{\frac{1}{3}}$$

(2)

$$\log_2^{4^x+15}=x+3\to 2^{x+3}=\underset{\hookrightarrow 2^{2x}}{4^x}+15\to 2^{x+3}-2^{2x}=15\to 2^x(2^3-2^x)=15$$
$$2^x=t$$
$$\to t(8-t)=15\to t^2-8t+15=0\to (t-3)(t-5)=0 \to t=3,\ t=5$$
$$\to 2^x=3\to x=\log_2^3$$
$$\to 2^x=5\to x=\log_2^5 \qquad \log_2^3+\log_2^5=\boxed{\log_2^{15}}$$

(3)

$$(\log_{21}^3)^2+\log_{21}^{147}\log_{21}^{1323}\to \log_{21}^{3^3\times7^2}=2\log_{21}^7+3\log_{21}^3$$
$$\log_{21}^{7^2\times3}=2\log_{21}^7+\log_{21}^3\to \log_{21}^{21}=\log_{21}^3+\log_{21}^7\xrightarrow{\log_{21}^3=a}\log_{21}^7=1-a$$
$$\to a^2+(2-2a+a)(2-2a+3a)=a^2+\underbrace{(2-a)(a+2)}_{4-a^2}=a^2-a^2+4=\boxed{4}$$

(4)

$$\log^{x^2-2x+1}+3\log(1-x)=5\to \log^{(x^2-2x+1)(1-x)^3}=5$$
$$\Rightarrow \log_{10}^{(1-x)^5}=5\to 10^5=(1-x)^5\to \overline{x=-9}\to \log_3^{-x}=\log_3^9=\boxed{2}$$

(5)

$$\log_2^{x^2+2x+4}+\log_2^{x-2}=3\Rightarrow \log_2^{(x-2)(x^2+2x+4)}=3\to \log_2^{x^3-8}=3 \ \hookrightarrow 8=x^3-8$$
$$\to x^3=16\to x=\sqrt[3]{16}=2^{\frac{4}{3}}$$
$$\to \log_{\sqrt[3]{2}}^x=\log_{2^{\frac13}}^{2^{\frac43}}=\boxed{4}$$

۶)

$$\log(2-x) - \log\frac{1}{(x-2)^2} = 3 \qquad \log_{10}\frac{2-x}{\frac{1}{(x-2)^2}} = 3 \longrightarrow (2-x)^3$$

$$10^3 = (2-x)^3 \longrightarrow \boxed{x=-8} \qquad \log_{\sqrt{2}} 8 = \boxed{6}$$

۷)

$$3^{x^2-2} = 3^{4x} \rightarrow x^2-2 = 4x \rightarrow x^2-4x-2=0 \rightarrow x = \frac{4\pm\sqrt{24}}{2} = 2\pm\sqrt{6}$$

$$\log_6^{x-2} = \log_6^{\sqrt{6}} = \boxed{\frac{1}{2}}$$

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۸)

$$\log_{18}^{8} = \frac{\log_3^8}{\log_3^{18}} = \frac{3\log_3^2}{2\log_3^3+\log_3^2} = \frac{\frac{15}{8}}{2+\frac{5}{8}} = \boxed{\frac{5}{7}}$$

۹)

$$\log_{12}^{6} = \frac{\log_4^6}{\log_4^{12}} = \frac{\log_4^3+\log_4^2}{\log_4^4+\log_4^3} = \frac{0.8+0.5}{1+0.8} = \boxed{\frac{13}{18}}$$

۱۰)

$$(a\log 2)x^2 + ax + b\log 2 = 0$$

یکی از ریشه‌ها (-1) است $\leftarrow a+c=b$

$$a\log 2 + b\log 2 = a \rightarrow a - a\log 2 = b\log 2$$

$$\rightarrow a\underbrace{(1-\log 2)} = b\log 2 \Rightarrow \frac{b}{a} = \frac{1-\log 2}{\log 2} = \frac{\log 5}{\log 2}$$

$$\log_{10}^{10} - \log_{10}^{2} = \log_{10}^{5}$$

$$\rightarrow \log_2^5$$

$$(\sqrt{2})^{\frac{b}{a}} = 2^{\frac{1}{2}\times\frac{b}{a}} = 2^{\frac{1}{2}\log_2^5} = 5^{\frac{1}{2}\log_2^2} = 5^{\frac{1}{2}} = \boxed{\sqrt{5}}$$