فرانک آقایی — تکلیف شماره‌ی ۲۴ — یازدهم تجربی

1

$y=x^2$: $\quad \xrightarrow{1} y=1 \rightarrow 3^{A+B}=1 \rightarrow A+B=0$

$\quad \xrightarrow{3} y=9 \rightarrow 3^{3A+B}=9 \rightarrow 3A+B=2$

$\Big\} \; 2A=2 \rightarrow A=1, \quad B=-1$

$f(x)=3^{x-1} \rightarrow 3^{x-1} \xrightarrow{x=0} 3^{-1}=\frac{1}{3}$

2

$2^{x+3}=4^x+15 \rightarrow 2^x\times 8=\underbrace{4^x}_{2^{2x}}+15 \rightarrow 2^x=t \rightarrow 8t=t^2+15$

$t^2-8t+15=0$

$(t-5)(t-3)=0 \rightarrow t=5$ یا $t=3 \rightarrow 2^x=5 \rightarrow x=\log_2 5$

$2^x=3 \rightarrow x=\log_2 3$

$\log_2 3+\log_2 5=\log_2 (3\times 5)=\log_2 15$

3

$21=3\times 7 \quad 147=3\times 7^2 \quad 1323=3^3\times 7^2 \quad \log_{21} 3=a \quad \log_{21} 7=b$

$\log_{21} 21=\log_{21} 3+\log_{21} 7=\boxed{a+b=1}$ / $\log_{21} 147=\log_{21} 3+2\log_{21} 7=\boxed{a+2b}$

$\log_{21} 1323=3\log_{21} 3+2\log_{21} 7=\boxed{3a+2b} \rightarrow a^2+(a+2b)(3a+2b)=4a^2+8ab+4b^2$

$4\underbrace{(a^2+2ab+b^2)}_{(a+b)^2}=4$

4

$x^2-2x+1>0 \rightarrow (x-1)^2>0 \rightarrow x>0 \quad x\neq 1 \quad 1-x>0 \rightarrow 1>x$

دامنه

$\log (x-1)^2+\underbrace{3\log (1-x)}_{\log (1-x)^3}=5 \rightarrow \log \left((1-x)^2\cdot(1-x)^3\right)=5$

$(1-x)^5=10^5 \rightarrow 1-x=10$

$x=-9$ در دامنه صدق می‌کند

$\log_3 9=2$

$x^2+2x+4>0 \to \Delta<0$ همواره مثبت $\to x-2>0 \to x>2$

$\log_2(x^2+2x+4) + \log_2(x-2) \Rightarrow \log_2 (x^2+2x+4)\cdot(x-2) = 3$ 5

$8 = x^3 - 8 \to x^3 = 16 \to x = \sqrt[3]{16} = 2\sqrt[3]{2} \to \log_{\sqrt[3]{2}} \sqrt[3]{16} = 4$

$\log (2-x) \underbrace{- \log \left(\frac{1}{(x-2)^2}\right)}_{+\log (x-2)^2} = 3 \to 2-x>0 \to 2>x,\ x\neq 2 \to 0>x$ دامنه $\to 0>x$ 6

$\log (2-x)\cdot(2-x)^2 = 3 \to 10^3 = (2-x)^3 \to 2-x = 10 \to x=-8$ ✓

$\log_{\sqrt{2}} -x \Rightarrow \log_{\sqrt{2}} 8 = 6$

$3^{(x^2-2)} = 3^{4x} \to x^2-4x-2=0 \to \Delta = 24 \to x = \frac{4\pm\sqrt{24}}{2} \to$ 7

$x = 2\pm\sqrt{6} \to \log_6 (2+\sqrt{6}-2) = \frac{1}{2}$ $\quad 2-\sqrt{6}$ غ‌ق‌ق $\quad x-2>0 \to x>2$

$\log_{18} 9 = \frac{\log_3 8}{\log_3 18} = \frac{\log_3 2^3}{\cancel{\log_3 3^2} \times \log_3 2} = \frac{3(\dots)}{2\times \dots} = \frac{3}{2}$ 8

$\log_{12} 6 \to \frac{\log_4 6}{\log_4 12} = \frac{\log_4 2 \times \log_4 3}{\log_4 2 \times \log_4 3} = \frac{1}{8}$ 9

$\log_4 4 = 1 \to \log_4 \frac{4}{2} = \log_4 4 - \log_4 2 = \log_4 2 \to 1 = 2\log_4 2$

$\log_4 2 = \frac{1}{2}$

10.

$$a\log 2 - a + b\log 2 = 0 \rightarrow a\log 2 + b\log 2 = a \rightarrow$$

$$\log 2\,(a+b) = a \rightarrow \log 2 = \frac{a}{a+b} \rightarrow \log_{2} 10 = \frac{a+b}{a} = 1+\frac{b}{a} \rightarrow$$

$$\frac{b}{a} = \log_{2} 10 - 1 \rightarrow \frac{b}{a} = \underbrace{\log_{2} 10 - \log_{2} 2}_{\log_{2} 5} \rightarrow \sqrt{2}^{\frac{b}{a}} = 2^{\frac{b}{2a}} =$$

$$2^{\frac{1}{2}\times\log_{2} 5} = 5^{\frac{1}{2}\times\overset{1}{\cancel{\log_{2} 2}}} = \sqrt{5}$$