1- $f(x) = 3^{Ax+B}$ $y = x^2$ $\begin{vmatrix} 1 \\ 1 \end{vmatrix} \begin{vmatrix} 3 \\ 9 \end{vmatrix}$

$f(1) = 3^{A+B} = 1 \Rightarrow A+B=0$
$f(3) = 3^{3A+B} = 9 = 3^2 \Rightarrow 3A+B=2$ $\Big\} \Rightarrow A=1,\ B=-1$ $f(0) = 3^{-1}$

2- $\log_2^{(4^x+15)} = x+3 \rightarrow 2^{2x}+15 = 2^{x+3} = \underbrace{2^x}_{t} \times 8$

$\hookrightarrow t^2 - 8t + 15 = 0 \rightarrow (t-3)(t-5) = 0$ — $2^x = 3$, $2^x = 5$ $\Big\}$ ⓵ $x_1 = \log_2^3$, $x_2 = \log_2^5$

$4^x + 15 > 0 \rightarrow$ ✓ قابل قبول ⓵

$\Rightarrow x_1 + x_2 = \log_2^3 + \log_2^5 = \log_2^{15}$

3- $(\log_{21}^3)^2 + \log_{21}^{147} \log_{21}^{1323}$ ⓸

$\log_{21}^{21} + \log_{21}^{7}$ $2\log_{21}^{21} + \log_{21}^{3}$ $\rightarrow \log_{21}^7 = 1 - \log_{21}^3 \Rightarrow (\log_{21}^3)^2 + (2-\log_{21}^3)(2+\log_{21}^3)$

$\rightarrow (\log_{21}^3)^2 + 4 - (\log_{21}^3)^2 = 4$

4- $\log(x^2-2x+1) + 3\log(1-x) = 5$ $\log_3^{(-x)}$ $1-x>0 \rightarrow 1>x$

$2\log(1-x)$ $\hookrightarrow 5\log(1-x) = 5 \Rightarrow \log(1-x) = 1 \Rightarrow 1-x = 10 \Rightarrow x = -9$

$\Rightarrow \log_3^9 = 2$

5- $\log_2^{x^2+2x+4} + \log_2^{(x-2)} = 3$ $\log_{\sqrt{2}}^x$ $x^2+2x+4 > 0$ ✓ $x>2$

$\log_2^{(x^3-8)} = 3 \longrightarrow x^3 - 8 = 8 \rightarrow x^3 = 16 \rightarrow x = \sqrt[3]{16} = 2^{\frac{4}{3}}$

$\longrightarrow \log_{\sqrt{2}}^{\sqrt[3]{16}} = 4$

6- $\log(2-x) - \log\frac{1}{(x-2)^2} = 3$ $\log_{\sqrt{2}}^{-x}$ $2>x$

$\log(2-x) - \log(2-x)^{-2} = 3 \rightarrow 3\log(2-x) = 3 \Rightarrow 2-x = 10 \rightarrow x = -8$

$\log_{\sqrt{2}}^{2^3} = 6$

7- $3^{x^2-2} = 81^x$ $\qquad \log_6^{(x-2)}$

$\downarrow$ 3^{4x}

$\rightarrow x^2-4x-2=0 \rightarrow x = \frac{4 \pm \sqrt{16+8}}{2} = 2 \pm \sqrt{6}$ $\begin{cases} 2-\sqrt{6} < 0 \quad \text{غ ق ق} \\ 2+\sqrt{6} > 0 \quad \text{ق ق} \end{cases}$

$\longrightarrow \log_6^{\sqrt{6}} = \frac{1}{2}$

8- $\log_3^2 = \frac{5}{8}$ $\qquad \log_{18}^{8}$

$\log_{18}^{8} = \frac{\log_3^8}{\log_3^{18}} = \frac{3\log_3^2}{\log_3^2+\log_3^9} = \frac{\frac{15}{8}}{\frac{21}{8}} = \frac{5}{7}$

($\log_3^2 = \frac{5}{8}$, $\log_3^9 = 2$)

9- $\log_4^3 = 0.8$ $\qquad \log_{12}^{6}$

$\downarrow$

$\log_2^3 = 1.6$

$\frac{\log_2^6}{\log_2^{12}} = \frac{1+\log_2^3}{2+\log_2^3} = \frac{2.6}{3.6} = \frac{13}{18}$

10- $(a\log 2)x^2 + ax + b\log 2 = 0$ $\qquad x_1 = -1$ $\qquad (\sqrt{2})^{\frac{b}{a}}$

$a\log 2 - a + b\log 2 = 0 \longrightarrow \log 2(a+b) - a = 0 \longrightarrow \left(\frac{a+b}{a}\right)\log 2 - 1 = 0 \longrightarrow \left(1+\frac{b}{a}\right)\log 2 = 1$

$\left(1+\frac{b}{a}\right) = \log_2^{10} \longrightarrow \log_2^5 = \frac{b}{a} \longrightarrow \sqrt{2}^{\log_2^5} = 5^{\log_2^{\sqrt{2}}} = 5^{\frac{1}{2}} = \sqrt{5}$