ترانه طاهریان _ تکلیف ۲۴ _ یازدهم دفتری

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$$y = 3^{Ax+B} \xrightarrow{(1,1)} 1 = 3^{A+B} \Rightarrow A+B=0$$

$$\xrightarrow{(3,9)} 9 = 3^{3A+B} \Rightarrow 3A+B=2 \quad \Big\} \Rightarrow$$

$$2A = 2 \Rightarrow A = 1 \ , \ B = -1$$

$$\xrightarrow{(0,y)} y = 3^{1\times 0 - 1} = 3^{-1} = \boxed{\frac{1}{3}}$$

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$$\log_2 (4^x + 15) = x + 3 \Rightarrow 2^{x+3} = 4^x + 15$$

$$4^x \to (2^2)^x = 2^{2x}$$

$$\Rightarrow 2^{x+3} - 2^{2x} = 15 \Rightarrow 2^x(2^3 - 2^x) = 15 \xrightarrow{2^x = t}$$

$$t(8-t) = 15 \Rightarrow t^2 - 8t + 15 = 0 \Rightarrow (t-3)(t-5) = 0$$

$$t = 3 \Rightarrow 2^x = 3 \Rightarrow x = \log_2 3 \qquad \log_2 3 + \log_2 5 = \boxed{\log_2 15}$$

$$t = 5 \Rightarrow 2^x = 5 \Rightarrow x = \log_2 5$$

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$$(\log_{21} 3)^2 + \log_{21}(147) \cdot \log_{21}(1323)$$

$$147 \to 49 \times 3 \qquad 1323 \to 27 \times 49$$

$$\log_{21} 3^3 \times 7^2 = 2\log_{21} 7 + 3\log_{21} 3$$

$$\log_{21} 7^2 \times 3 = 2\log_{21} 7 + \log_{21} 3$$

$$\log_{21} 21 \ (21 \to 3\times 7) = \log_{21} 3 + \log_{21} 7 \xrightarrow{\log_{21} 3 = a} \log_{21} 7 = 1 - a$$

$$\Rightarrow a^2 + (2 - 2a + a)(2 - 2a + 3a) = a^2 + (2-a)(a+2)$$

$$(2-a)(a+2) = 4 - a^2$$

$$= a^2 + 4 - a^2 = \boxed{4}$$

$$\log(x^2-2x+1) + 3\log(1-x) = 5 \Rightarrow \log(x^2-2x+1)(1-x)^3 = 5 \quad -4$$

$$(x^2-2x+1) = (x-1)^2 \qquad 3\log(1-x) = \log(1-x)^3$$

$$\Rightarrow \log_{10}(1-x)^5 = 5 \Rightarrow 10^5 = (1-x)^5 \Rightarrow 1-x = 10 \Rightarrow$$

$$x = -9 \qquad \log_3 (-x) = \log_3 9 = \boxed{2}$$

$$\log_2(x^2+2x+4) + \log_2(x-2) = 3 \Rightarrow \log_2 (x-2)(x^2+2x+4) = 3 \quad -5$$

$$\Rightarrow \log_2 (x^3 + 2x^2 + 4x - 2x^2 - 4x - 8) = 3 \Rightarrow x^3 - 8 = 8$$

$$\Rightarrow x^3 = 16 \Rightarrow x = \sqrt[3]{16} = 2^{\frac{4}{3}}$$

$$\log_{\sqrt[3]{2}} x = \log_{2^{\frac{1}{3}}} 2^{\frac{4}{3}} = \boxed{4}$$

$$\log(2-x) - \log\frac{1}{(x-2)^2} = 3 \Rightarrow \log_{10} \frac{2-x}{\frac{1}{(x-2)^2}} = 3 \quad -6 \qquad \frac{2-x}{\frac{1}{(x-2)^2}} = (2-x)^3$$

$$\Rightarrow 10^3 = (2-x)^3 \Rightarrow 2-x = 10 \Rightarrow x = -8 \qquad \log_{\sqrt{2}} (-x) = \log_{\sqrt{2}} 8 = \boxed{6}$$

$$3^{x^2-2} = (3^4)^x = 3^{4x} \Rightarrow x^2 - 2 = 4x \Rightarrow x^2 - 4x - 2 = 0 \quad -7$$

$$x = \frac{4 \pm \sqrt{16+8}}{2} = \frac{4 \pm 2\sqrt{6}}{2} = 2 \pm \sqrt{6} \qquad \log_6 (x-2) =$$

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$$\log_6 \pm\sqrt{6} = \boxed{\frac{1}{2}}$$

s.a.m

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$$\log_{۱۸}^{۸} = \frac{\log_{۳}^{۸}}{\log_{۳}^{۱۸}} = \frac{۳\log_{۳}^{۲}}{۲\log_{۳}^{۳} + \log_{۳}^{۲}} = \frac{\frac{۱۵}{۸}}{۲+\frac{۵}{۸}}$$

($۸ \to ۲^{۳}$ ، $۱۸ \to ۳^{۲}\times ۲$ ، $\log_{۳}^{۳} = ۱$)

$$= \frac{۱۵}{۲۱} = \boxed{\frac{۵}{۷}}$$

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$$\log_{۱۲}^{۶} = \frac{\log_{۴}^{۶}}{\log_{۴}^{۱۲}} = \frac{\log_{۴}^{۳} + \log_{۴}^{۲}}{\log_{۴}^{۴} + \log_{۴}^{۳}} = \frac{۰/۸ + ۰/۵}{۱ + ۰/۸} = \frac{۱/۳}{۱/۸} = \boxed{\frac{۱۳}{۱۸}}$$

($۶ \to ۳\times ۲$ ، $۱۲ \to ۳\times ۴$)

۱۰-

$$(a\log^{۲})x^{۲} + ax + b\log^{۲} = ۰$$

یکی از ریشه‌ها ۱ است $\Leftarrow a+c=b$

$$a\log^{۲} + b\log^{۲} = a \Rightarrow a - a\log^{۲} = b\log^{۲}$$

$$\Rightarrow a(۱-\log^{۲}) = b\log^{۲} \Rightarrow \frac{b}{a} = \frac{۱-\log^{۲}}{\log^{۲}} = \frac{\log^{۵}}{\log^{۲}}$$

$$\log_{۱۰}^{۱۰} - \log_{۱۰}^{۲} = \log_{۱۰}^{۵}$$

$$(\sqrt{۲})^{\frac{b}{a}} = ۲^{\frac{۱}{۲}\times\frac{b}{a}} = ۲^{\frac{۱}{۲}\log_{۲}^{۵}} = ۵^{\frac{۱}{۲}\log_{۲}^{۲}} = ۵^{\frac{۱}{۲}} = \boxed{\sqrt{۵}}$$

($\frac{b}{a} = \log_{۲}^{۵}$)

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