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نام و نام خانوادگی ....[illegible]........ پاسخنامه تشریحی تکلیف شماره ..۲۶.... کلاس ....[illegible]....

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$f(x)=3^{Ax+B}$, $y=x^2 \to$ $x=1, 3$ محل تلاقی

$3^{Ax+B}=x^2 \xrightarrow{x=1} 3^{A+B}=1 \Rightarrow 3^{A+B}=3^0 \Rightarrow A+B=0$

$\xrightarrow{x=3} 3^{3A+B}=9 \Rightarrow 3^{3A+B}=3^2 \Rightarrow 3A+B=2$

$2A=2 \to A=1$, $\to B=-1$

$f(x)=3^{x-1} \Rightarrow x=0 \Rightarrow f(x)=3^{-1} \Rightarrow f(x)=\frac{1}{3}$ جواب

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$\log_2(4^x+15)=x+3 \Rightarrow 2^{x+3}=4^x+15 \Rightarrow 2^{x+3}=2^{2x}+15 \Rightarrow 2^{x+3}-2^{2x}=15 \Rightarrow 2^x(2^3-2^x)=15$

$\xrightarrow{2^x=t}$ $t(8-t)=15 \Rightarrow -t^2+8t=15 \Rightarrow 15+t^2-8t=0 \Rightarrow (t-3)(t-5)=0$; $t=3 \to 2^x=3 \Rightarrow x=\log_2 3$; $t=5 \to 2^x=5 \Rightarrow x=\log_2 5$

$\alpha+\beta=\log_2 3+\log_2 5=\boxed{\log_2 15}$ → جواب

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$(\log_{21}3)^2+\log_{21}(\overbrace{147}^{3\times 49})\log_{21}(\overbrace{1323}^{3^3\times 7^2}) \Rightarrow (\log_{21}3)^2+(\log_{21}7^2+\log_{21}3)\times(\log_{21}3^3+\log_{21}7^2) \Rightarrow$

$(\underbrace{\log_{21}3}_{t})^2+(2\underbrace{\log_{21}7}_{(1-t)}+\log_{21}3)\times(3\log_{21}3+2\underbrace{\log_{21}7}_{1-t}) \Rightarrow t^2+(2-2t+t)(3t+2-2t)=t^2+(2-t)(2+t)$

$\Rightarrow t^2+4-t^2=4$ → جواب

$\log_{21}7=\log_{21}21-\log_{21}7=\log_{21}3 \to \log_{21}7=1-t$

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$\log(x^2-2x+1)+3\log(1-x)=5 \Rightarrow \log(1-x)^2+3\log(1-x)=5 \Rightarrow 2\log(1-x)+3\log(1-x)=5$

$\Rightarrow 5\log(1-x)=5 \Rightarrow \log_{10}(1-x)=1 \Rightarrow 1-x=10 \Rightarrow -x=9 \Rightarrow x=-9$

$\log_3(-x) = \log_3(-(-9)) = \log_3 9 = 2$ جواب

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$\log_2(x^2+2x+4)+\log_2(x-2)=3 \Rightarrow \log_2(x^2+2x+4)(x-2)=\log_2 8 \Rightarrow (x^2+2x+4)(x-2)=8 \Rightarrow$

$x^3-\cancel{2x^2}+\cancel{2x^2}-\cancel{4x}+\cancel{4x}-8=8 \Rightarrow x^3=16 \Rightarrow x=\sqrt[3]{16}$

$\log_{\sqrt[3]{2}} x \Rightarrow \log_{2^{\frac{1}{3}}} 16^{\frac{1}{3}} \Rightarrow 1\log_2 16 = 1\times 4=4$ جواب

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$\log(2-x) - \log\frac{1}{(2-x)^2} = 3 \Rightarrow \log_{10}^{(2-x)\times(2-x)^2} = \log_{10}(2-x)^3 = 3$

$\Rightarrow (2-x)^3 = 10^3 \Rightarrow 2-x=10 \Rightarrow x=-8$ (*)

$\log_{\sqrt{2}}(-x) = \log_{2^{\frac{1}{2}}} 2^3 = 6\log_2^2 = 6\times 1 = 6$ ← جواب

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$3^{x^2-2} = 81^x \Rightarrow 3^{x^2-2} = 3^{4x} \Rightarrow x^2-2 = 4x \Rightarrow x^2-4x-2=0 \rightarrow x = \frac{-b\pm\sqrt{\Delta}}{2a} \Rightarrow$

$\Delta = b^2-4ac = 16-(\underbrace{4\times 1\times -2}_{-8}) = 24 \; , \; \sqrt{\Delta} = \sqrt{24} = 2\sqrt{6} \Rightarrow x = \frac{4\pm 2\sqrt{6}}{2}$ → $x = 2+\sqrt{6}$ ✓ / $x = 2-\sqrt{6}$ غ ق ق

$\log_6^{(x-2)} = \log_6^{(\cancel{2}+\sqrt{6}-\cancel{2})} = \log_6^{\sqrt{6}} = \log_6^{6^{\frac{1}{2}}} = \frac{1}{2}\underbrace{\log_6^6}_{=1} = \frac{1}{2}$ ← جواب

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$\log_3^2 = \frac{5}{8} \; , \; \log_{18}^8 = ? \rightarrow \log_2^3 = \frac{8}{5}$ (*)

$\log_{18}^8 \Rightarrow \frac{\log_2^8}{\log_2^{18}} = \frac{\log_2^{8 \to 2^3}}{\log_2^{18 \to 9\times 2}} = \frac{3}{\log_2^9 + \log_2^2} = \frac{3}{\underbrace{2\log_2^3}_{(*)\; 2\times\frac{8}{5}} + 1} = \frac{3}{\frac{16}{5}+1} = \frac{3}{\frac{21}{5}} = \cancel{3}\times\frac{5}{\cancel{21}_7} = \frac{5}{7}$ ← جواب

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$\log_4^3 = 0.8 \Rightarrow \log_3^4 = \frac{10}{8} \Rightarrow 2\log_3^2 = \frac{10}{8} \Rightarrow \log_3^2 = \frac{5}{8} \; , \; \log_3^4 = \frac{\cancel{10}^5}{\cancel{8}_4}$

$\log_{12}^6 = \frac{\log_3^{6 \to 2\times 3}}{\log_3^{12 \to 3\times 4}} = \frac{\overbrace{\cancel{\log_3^2}}^{\frac{5}{8}} + \overbrace{\cancel{\log_3^3}}^{1}}{\underbrace{\cancel{\log_3^3}}_{1} + \underbrace{\cancel{\log_3^4}}_{\frac{5}{4}}} = \frac{\frac{13}{8}}{\frac{9}{4}} = \frac{13\times\cancel{4}}{\cancel{8}_2\times 9} = \frac{13}{18}$ ← جواب

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$(a\log 2)x^2 + ax + b\log 2 = 0 \xRightarrow{x=-1} (a\log 2) - a + b\log 2 = 0 \Rightarrow b\log 2 = a - (a\log 2)$

$\Rightarrow b\log 2 = a(1-\log 2) \Rightarrow \frac{b}{a} = \frac{1-\log 2}{\log 2} = \frac{\log_{10}^{10} - \log_{10}^2}{\log 2} = \frac{\log_{10}^{\frac{10}{2}}}{\log_{10}^2} = \log_2^5$ (*)

$(\sqrt{2})^{\frac{b}{a}} \overset{(*)}{\Rightarrow} (\sqrt{2})^{\log_2^5} \Rightarrow 5^{\log_2^{2^{\frac{1}{2}}}} = 5^{\frac{1}{2}} = \sqrt{5}$ ← جواب

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