①

$x=1 \to 1=3^{A+B}$ $\qquad A+B=0$

$x=3 \to 9=3^{3A+B}$ $\qquad 3A+B=2$

$2A=2 \to A=1$

$B=-1$

تلاقی $3^{-1} \Rightarrow \frac{1}{3}$

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②

$\log_2^{(4^x+15)} = x+3$

$4^x+15=2^{x+3}$

$2^{2x}-2^{x+3}+15=0$

$(2^x)^2-8\times 2^x+15=0 \to 2^x=t$

$t^2-8t+15=0 \to (t-3)(t-5)=0$

$t=3 \to 2^x=3 \qquad x=\log_2^3$

$t=5 \to 2^x=5 \qquad x=\log_2^5$

$\log_2^3+\log_2^5=\boxed{\log_2^{15}}$

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③

$(\log_{21}^3)^2+(\log_{21}^7+\log_{21}^{21})(\log_{21}^3+2\log_{21}^{21})$ $\qquad \log_{21}^7 = 1-\log_{21}^3$

$(\log_{21}^3)^2+(2-\log_{21}^3)(2+\log_{21}^3)=(\log_{21}^3)^2+4-(\log_{21}^3)^2=\boxed{4}$

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④

$\log^{(1-x)^2}+3\log^{1-x}=5$ $\qquad \log_3^9=\boxed{2}$

$5\log(1-x)=5$

$\log(1-x)=1 \qquad (1-x)=10 \qquad \boxed{x=-9}$

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⑤

$\log_2^{x^2+2x+4}+\log_2^{x-2}=\log_2^{x^3-8}=3$

$x^3-8=8 \to x^3=16 \to x=\sqrt[3]{16}$ $\qquad \log_{\sqrt[3]{2}\to 2^{\frac{1}{3}}}^{\sqrt[3]{16}\to 2^{\frac{4}{3}}} \Rightarrow \frac{\frac{4}{3}}{\frac{1}{3}}=\boxed{4}$

⑥

$\log^{2-x} - \log^{(2-x)^{-2}} = 3 \Rightarrow 3\log^{2-x} = 3 \sim \log^{2-x} = 1$

$2-x = 1 \quad x = -1 \qquad \log_{\sqrt{3}}^{8} = \frac{3}{\frac{1}{2}} = 6$

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⑦

$3^{x^2-2} = 3^{4x}$

$x^2-4x-2=0 \rightarrow (x-2)^2 = 6 \Rightarrow x-2 = \sqrt{6} \Rightarrow x = 2+\sqrt{6}$

$x-2 = -\sqrt{6} \Rightarrow x = 2-\sqrt{6}$ غ ق ق

$\log_{6}^{2+\sqrt{6}-2} = \log_{6}^{\sqrt{6}} = \frac{1}{2}$

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⑧

$\log_{18}^{8} = \frac{\log_2^8}{\log_2^{18}} = \frac{3}{\log_2^9 + \log_2^2} = \frac{3}{1+2\log_2^3} = \frac{3}{1+2(\frac{5}{8})}$

$\frac{5}{8}$ تقریبی

$\frac{3}{\frac{21}{5}} = 3 \times \frac{5}{21} = \boxed{\frac{5}{7}}$

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⑨

$\log_{12}^{6} = \frac{\log_2^6}{\log_2^{12}} = \frac{\log_2^3 + 1}{\log_2^3 + 2} = \frac{1.6+1}{1.6+2} \Rightarrow$

$\frac{1}{2}\log_2^3 = \frac{8}{10}$

$\log_2^3 = 1.6$

$\frac{2.6}{3.6} \Rightarrow \boxed{\frac{13}{18}}$

$x=-1 \rightarrow a\log 2 - a + b\log 2 = 0 \rightarrow \log 2 = y \quad ay - a + by = 0$ (1)

$\div a \rightarrow y - 1 + \frac{b}{a}y = 0 \rightarrow y + \frac{b}{a}y = 1 \rightarrow 1 - \frac{b}{a} = \frac{1}{\log 2}$

$1 - \frac{b}{a} = \log_2 10 \rightarrow 1 - \frac{b}{a} = 1 + \log_2 5 \Rightarrow \frac{b}{a} = -\log_2 5$

$\sqrt{2}^{-\log_2 5} = 5^{-\frac{1}{2}} = \sqrt{\frac{1}{5}}$