بنام خداوند ...

پریناجانقربان
اسم نوشته شود

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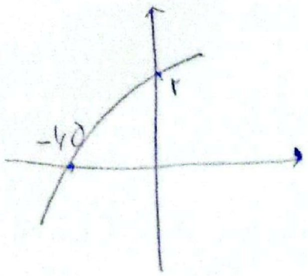
$y = 1 - \log_c (ax - b)$

$b + c = -\frac{3}{2}$ $\qquad (a+c)b$



$(0,2) \rightarrow 1 - \log_c^{-b} = 2 \Rightarrow -\log_c^{-b} = 1 \Rightarrow c^{-1} = -b$

$\frac{1}{c} = -b \Rightarrow c = -\frac{1}{b}$

$b - \frac{1}{b} = -\frac{3}{2}$

$\Rightarrow \frac{b^2 - 1}{b} = -\frac{3}{2} \Rightarrow b^2 + \frac{3}{2}b - 1 = 0 \xrightarrow{\times 2} 2b^2 + 3b - 2 = 0 \Rightarrow b^2 + 3b - 4 = 0$

$(b+4)(b-1) = 0$

$\frac{-4}{2} = -2 \,,\ \frac{1}{2}$

$b = -2 \,,\ \frac{1}{2}$ غلط

$c = \frac{1}{2} \,,\ -2$ $\quad c > 0$

$(-\frac{3}{2}, 0) \rightarrow 1 - \log_{\frac{1}{2}}^{(-\frac{3}{2}a + 2)} = 0 \Rightarrow \frac{1}{2} = -\frac{3}{2}a + 2 \Rightarrow -\frac{3}{2}a = -\frac{3}{2} \Rightarrow \boxed{a = 1}$

$(a+c)b = (1 + \frac{1}{2})(-2) = (\frac{3}{2})(-2) = -3$

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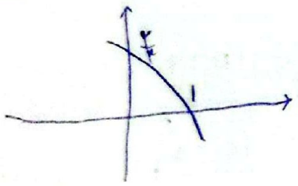
$f(x) = 1 + c \times 3^{a+bx}$ $\qquad f(-1) = ?$



$(0, \frac{2}{3}) \rightarrow 1 + c \times 3^a = \frac{2}{3} \Rightarrow -\frac{1}{3} = c \times 3^a$

$(1, 0) \rightarrow 1 + c \times 3^{a+b} = 0 \Rightarrow c \times 3^{a+b} = -1$

$\div \qquad 3 = \frac{c \times 3^a \times 3^b}{c \times 3^a}$

$\Rightarrow 3^b = 3 \Rightarrow \boxed{b = 1}$

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$c \times 3^a = -\frac{1}{3}$

$c \times 3^{a+1} = -1 \xrightarrow{\div 9} \frac{c \times 3^{a+1}}{3^2} = c \times 3^{a-1} = -\frac{1}{9}$

$f(-1) = 1 + \underbrace{c \times 3^{a-1}}_{-\frac{1}{9}} = -\frac{1}{9} + 1 = \boxed{\frac{5}{9}}$

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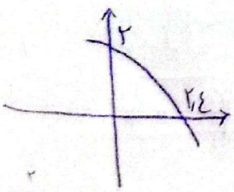
$y = c + \log_5 (ax + b)$ $\qquad \frac{a}{b}$



$(0,2) \rightarrow c + \log_5^b = 2 \Rightarrow \log_5^b = 2 - c \Rightarrow \boxed{b = 5^{2-c}}$

$(2.4, 0) \rightarrow c + \log_5^{2.4a+b} = 0 \Rightarrow \boxed{5^{-c} = 2.4a + b}$ $\quad \div$

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$\frac{5^{2-c}}{5^{-c}} = \frac{b}{2.4a + b}$

$\Rightarrow 25 = \frac{b}{2.4a + b}$

$25 \times 2.4 = 60.0$

$\Rightarrow 60a + 25b = b \Rightarrow 60a = -24b$

$\frac{a}{b} = \frac{-24}{60} = \frac{-4}{10} = \boxed{-\frac{2}{5}}$

بنام خداوند دانش

④ $\log_{3}(|x^2-2|-x)$ $\quad |x^2-2|-x>0$

$x^2-2=0$
$(x+\sqrt{2})(x-\sqrt{2})=0$
$x=\pm\sqrt{2}$

$x^2-2-x>0 \quad x^2-2-x=(x-2)(x+1)=0, \quad x=2,-1$

$-x^2+2-x>0$
$x^2+x-2<0$
$(x+2)(x-1)$
$-2 \quad 1$

$(-\infty,-\sqrt{2}] \cup (2,+\infty)$
$\cup(-\sqrt{2},1) = (-\infty,1)\cup(2,+\infty)$

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⑤ $f^{-1}(10)=-1 \Rightarrow f(-1)=10$
$f(x)=2+2^{b-ax} \qquad g(x)=-x^2-3x+8 \qquad (1,4)$
$2b-a$
$\Rightarrow -1-3+8=4$
$(1,4) \Rightarrow 4=2+2^{b-a} \Rightarrow 2=2^{b-a} \Rightarrow b-a=1$
$(-1,10) \Rightarrow 2+2^{b+a}=10 \Rightarrow 2^{b+a}=2^3 \Rightarrow b+a=3$ $\Big\} \Rightarrow 2b=4$ $\boxed{b=2}$ $\boxed{a=1}$
$2\times2-1=\boxed{3}$

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⑥ $(2,4)\ (1,0) \quad y=x^2-x \qquad f(x)=-2+\left(\frac{1}{2}\right)^{Ax+B}$
$\to 1-1=0$
$\to 4-2=2$
$f(2)=-2+\left(\frac{1}{2}\right)^{-x}=-2+\left(\frac{1}{2}\right)^{-2} \quad f(2)=\boxed{2}$
$(1,0) \Rightarrow -2+\left(\frac{1}{2}\right)^{A+B}=0 \Rightarrow \left(\frac{1}{2}\right)^{A+B}=2 \to \boxed{A+B=-1}$
$(2,2) \Rightarrow -2+\left(\frac{1}{2}\right)^{2A+B}=2 \Rightarrow \left(\frac{1}{2}\right)^{2A+B}=4=2^2 \Rightarrow \boxed{2A+B=-2}$
$\boxed{-1=A} \quad \boxed{B=0}$

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⑦ هر ساعت $\frac{1}{9}$ از دست بدهد
چند دقیقه $\frac{1}{6}$ باقی بماند
$m_0\left(\frac{8}{9}\right)^{\frac{t}{60}}$ دقیقه $\to$ $\quad \log_5 2 \simeq 1/4 \quad \log_5 3 \simeq 2/4$
$m_0\left(\frac{8}{9}\right)^{\frac{t}{60}}=\frac{1}{6}m_0$
$\left(\frac{8}{9}\right)^{\frac{t}{60}}=\frac{1}{6} \xRightarrow{\log_5} \frac{t}{60}\log_5\frac{8}{9}=\log_5\frac{1}{6}$

به توان دو رسیده است

$\frac{t}{60}(3\log_5^2 - 2\log_5^3) = -\log_5^6 \Rightarrow \frac{t}{60}\left(\frac{3\times5}{12} - \frac{2\times5}{7}\right) = -\frac{95}{84}$

$\log_5^2 = \frac{1}{\log_2^5} = \frac{1}{2.4} = \frac{1}{\frac{24}{10}} = \frac{10}{24} = \frac{5}{12}$ $\quad \frac{5}{12} - \frac{10}{7}$

$\log_5^3 = \frac{1}{\log_3^5} = \frac{1}{1.4} = \frac{1}{\frac{14}{10}} = \frac{10}{14} = \frac{5}{7}$ $\quad = \frac{105-120}{84} = \frac{-15}{84}$

$\log_5^6 = \log^2 + \log^3 = \frac{5}{12} + \frac{5}{7} = \frac{35+60}{84} = \frac{95}{84}$

$\left(\frac{t}{60}\right)\left(\frac{-15}{84}\right) = -\frac{95}{84} \Rightarrow \boxed{t = 380\ min}$

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⑧ هر هفته ۲٫۵ درصد از دست

$\frac{875}{1000} = \frac{35}{40} = \frac{7}{8}$

$\log_2^3 \simeq 1.6 \quad \log_7^3 \simeq 0.6$

$m_0\left(\frac{875}{1000}\right)^{\frac{t}{7}}$ (روز) $\Rightarrow m_0\left(\frac{7}{8}\right)^{\frac{t}{7}} \Rightarrow m_0\left(\frac{7}{8}\right)^{\frac{t}{7}} = \frac{1}{7}m_0$ (صفر روز) $\frac{1}{7}$

$\xRightarrow{\log_3} \frac{t}{7}\log_3^{\frac{7}{8}} = \log_3^{\frac{1}{7}} = -\log_3^7$

$\Rightarrow \frac{t}{7}(\log_3^7 - 3\log_3^2) = -\log_3^7$

$\frac{t}{7}\left(\frac{5}{3} - \frac{3\times5}{8}\right) = -\frac{5}{3} \Rightarrow \frac{t}{7}\left(\frac{-5}{24}\right) = -\frac{5}{3}$

$\frac{40-45}{24} = \frac{-5}{24}$

$\boxed{t = 56\ \text{روز}} \rightarrow$ ۸ هفته

$\log_3^7 = \frac{1}{\log_7^3} = \frac{1}{0.6} = \frac{1}{\frac{6}{10}} = \frac{10}{6} = \boxed{\frac{5}{3}}$

$\log_3^2 = \frac{1}{\log_2^3} = \frac{1}{\frac{16}{10}} = \frac{10}{16} = \boxed{\frac{5}{8}}$

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⑨

$P_0\left(\frac{96}{100}\right)^{t}$ (روز) $= \frac{1}{3}P_0$

$\xRightarrow{\log_{10}} t\log_{10}^{\frac{96}{100}} = \log_{10}^{\frac{1}{3}} \Rightarrow t(\underbrace{\log^{96} - \log_{10}^{100}}_{-0.02}) = \underbrace{-\log_{10}^3}_{-0.48}$

$\log^{96} = 5\log^2 + \log^3 = (5\times0.3 + 0.48) = 1.98$ $\quad (1.5 + 0.48)$

$\Rightarrow 2t = 48 \Rightarrow \boxed{t = 24\ \text{روز}}$

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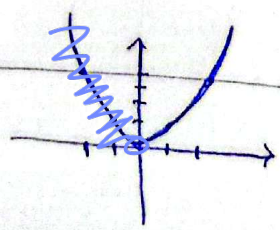
⑩

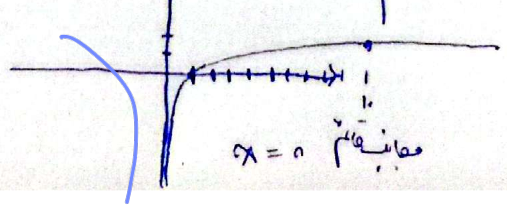
الف) $y = 9^{\log_3^x} \rightarrow y = x^{\log_3^9} \Rightarrow y = x^2$

$D = (0, +\infty)$



ب) $y = \log^{x^2} - 2\log_{10}^x$

$D = \mathbb{R} - \{0\}$



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