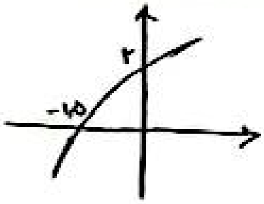
باران کاظمی — تمرین شماره ۲۵

(1



$y = 1 - \log_c^{(ax-b)}$ $\qquad b + c = -\frac{3}{2} \to b = -\frac{3}{2} - c$

$\xrightarrow{x=0} 1 - \log_c^{-b} = 2 \to \log_c^{-b} = -1 \to \frac{1}{c} = -b \to \frac{1}{c} = \frac{3}{2} + c$

$1 = \frac{3}{2}c + c^2 \to c^2 + \frac{3}{2}c - 1 = 0$

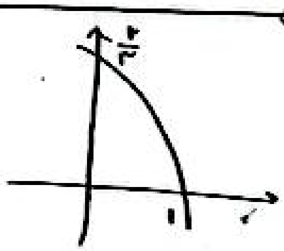
$\dfrac{-\frac{3}{2} \pm \sqrt{\frac{9}{4} + 4}}{2} = \dfrac{-\frac{3}{2} \pm \frac{5}{2}}{2} \to -\frac{4}{2},\ \frac{1}{2} \to \boxed{c = \frac{1}{2}}$ $\qquad \dfrac{-\frac{3}{2} - \frac{5}{2}}{2}$ غ ق ق $\quad c > 0$

$b = -\frac{3}{2} - \frac{1}{2} = \boxed{-2}$ $\quad \xrightarrow{x=-1.5} 0 = 1 - \log_{\frac{1}{2}}^{(ax+2)} \to \log_{\frac{1}{2}}^{ax+2} = 1 \to \frac{1}{2} = -1.5a + 2$

$1.5a = 1.5 \to \boxed{a = 1}$

$(a+c)b = (\underbrace{1 + \frac{1}{2}}_{\frac{3}{2}})(-2) = \boxed{-3}$

(2



$f(x) = 1 + c \times 3^{a+bx}$ $\qquad f(-1) = ?$

$0 = 1 + c \times 3^{a+b} \to 0 = 1 + \underbrace{c \times 3^a}_{-\frac{1}{3}} \times 3^b \to \frac{1}{3} \times 3^b = 1$

$0 = 1 + c \times 3^{a+1}$ $\qquad 3^b = 3 \to \boxed{b = 1}$

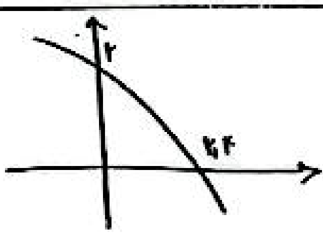
$\frac{2}{3} = 1 + c \times 3^a \to c \times 3^a = -\frac{1}{3}$

$\left.\begin{array}{l} f(-1) = y = 1 + c \times 3^{a-1} \\ f(1) = 0 = 1 + c \times 3^{a+1} \end{array}\right\} \xrightarrow{\div} \frac{-1}{y-1} = \frac{\cancel{c} \times 3^{a+1}}{\cancel{c} \times 3^{a-1}} \to \frac{-1}{y-1} = 3^2$

$-1 = 9y - 9 \to y = \frac{8}{9}$

$y = \boxed{f(-1) = \frac{8}{9}}$

(3



$y = c + \log_5^{(ax+b)}$ $\qquad \frac{a}{b} = ?$

$\left\{\begin{array}{l} 2 = \cancel{c} + \log_5^{b} \\ 0 = \cancel{c} + \log_5^{2.4a+b} \end{array}\right.$

$2 = \log_5^{b} - \log_5^{2.4a+b} \to 2 = \log_5^{\frac{b}{2.4a+b}}$

$25 = \frac{b}{2.4a+b} \to 25b + 60a = b \to 24b = -60a \to \boxed{\frac{a}{b} = -\frac{2}{5}}$

(4

$f(x) = \log_2^{(|x^2-2| - x)}$ $\qquad |x^2-2| - x > 0 \to |x^2-2| > x \to -x < x^2 - 2 > x$

$\to x^2 - 2 > x \to x^2 - x - 2 > 0 \to (x-2)(x+1) > 0$ $\quad \begin{array}{c} -1 \quad 2 \\ + \mid - \mid + \end{array}$

$(-\infty, -1) \cup (2, +\infty)$ ①

$\to x^2 - 2 < -x \to x^2 + x - 2 < 0 \to (x+2)(x-1) < 0$ $\quad \begin{array}{c} -2 \quad 1 \\ + \mid - \mid + \end{array}$

$(-2, 1)$ ②

①∩② $(-2, -1)$

$f(x) = ۲ + ۲^{b-ax}$

$g(x) = -x^۲ - ۳x + ۸$

$f^{-1}(۱۰) = -۱ \to f(-۱) = ۱۰ \to ۱۰ = ۲ + ۲^{b+a}$

$۲ + ۲^{b-a} = -۱ - ۳ + ۸$ (۵

$۲^{b-a} = ۲ \to \begin{cases} b-a=۱ \\ b+a=۳ \end{cases} \to a=۱$

$۲b = ۴ \to b = ۲$

$۲b - a = ۴ - ۱ = \boxed{۳}$

$f(x) = -۲ + \left(\frac{۱}{۲}\right)^{Ax+B}$ (۶

$y = x^۲ - x$

$۰ = -۲ + \left(\frac{۱}{۲}\right)^{A+B} \to \left(\frac{۱}{۲}\right)^{A+B} = ۲ \to A+B = -۱$

$۲ = -۲ + \left(\frac{۱}{۲}\right)^{۲A+B} \to ۴ = \left(\frac{۱}{۲}\right)^{۲A+B} \to ۲A+B = -۲$

$\begin{cases} ۲A+B = -۲ \\ A+B = -۱ \end{cases}$

$A = -۱ \ , \ B = ۰$

$f(x) = -۲ + \left(\frac{۱}{۲}\right)^{-x}$

$\to f(۳) = -۲ + \cancel{\left(\frac{۱}{۲}\right)^{-۳}}^{۸} = \boxed{۶}$

$\log_۵^۲ = \frac{۱۰}{۲۴} = \frac{۵}{۱۲} \qquad \log_۵^۳ = \frac{۱۰}{۱۴}$ (۷

$m(t) = \left(\frac{۸}{۹}\right)^t \to \left(\frac{۸}{۹}\right)^t = \frac{۱}{۶}$

$\log_۵ \left(\frac{۸}{۹}\right)^t = \log_۵ \frac{۱}{۶} \to t \log_۵ \frac{۸}{۹} = \log_۵ \frac{۱}{۶} \to t(۳\log_۵^۲ - \log_۵^۹) = -\log_۵^۶$

$t\left(\frac{۵}{۴} - \left(۲\left(\frac{۱۰}{۱۴}\right)\right)\right) = -\left(\underbrace{\frac{۵}{۱۲} + \frac{۱۰}{۱۴}}_{\frac{۷۰+۱۲۰}{۱۶۸} = \frac{۱۹۰}{۱۶۸}}\right)$

$t\left(\underbrace{\frac{۵}{۴} - \frac{۱۰}{۷}}_{\frac{۳۵-۴۰}{۲۸} = -\frac{۵}{۲۸}}\right) = -\frac{۱۹۰}{۱۶۸} \to t\left(-\frac{۵}{۲۸}\right) = -\frac{۱۹۰}{۱۶۸} \to t = \frac{۳۸}{۶} h \xrightarrow{\times ۶۰}$

$\frac{۳۸}{۶} \times ۶۰ = \boxed{۳۸۰ \text{ min}}$

$m(t) = \left(\frac{۸۷٫۵}{۱۰۰}\right)^t \to \left(\frac{۷}{۸}\right)^t = \frac{۱}{۷}$ (۸

$\log_۷^۳ = ۰٫۶ \to \log_۲^۷ = \frac{۵}{۳}$

$\log_۲^۳ = ۱٫۶ \to \log_۲^۳ = \frac{۵}{۸}$

$\log_۳ \left(\frac{۷}{۸}\right)^t = \log_۳ \frac{۱}{۷} \to t(\log_۲^۷ - ۳\log_۲^۲) = \log_۲^۱ - \log_۳^۷$

$t\left(\underbrace{\frac{۵}{۳} - \frac{۱۵}{۸}}_{\frac{۴۰-۴۵}{۲۴}}\right) = ۰ - \frac{۵}{۲} \to \cancel{-}\frac{\cancel{۵}}{۲۴} t = \cancel{-}\frac{\cancel{۵}}{۲} \to t = ۱۸ \ \text{min} \xrightarrow{\times ۷} \boxed{۵۴ \text{ in}}$

$m(t) = \left(\frac{۹۶}{۱۰۰}\right)^t \to \left(\frac{۹۶}{۱۰۰}\right)^t = \frac{۱}{۲} \to \log\left(\frac{۹۶}{۱۰۰}\right)^t = \log \frac{۱}{۲}$ $۹۶ = ۲\times۲\times۲\times۲\times۲\times۳$ (۹

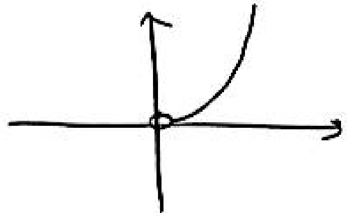
$t(\log ۹۶ - \cancel{\log ۱۰۰}^{۲}) = \cancel{\log ۱} - \cancel{\log ۲}^{۰٫۳}$

$t(\underbrace{۱٫۵ + ۰٫۴۸ - ۲}_{-۰٫۰۲}) = -۰٫۱۴۸ \to \boxed{t = ۲۴ \text{ in}}$

(10

الف) $y = 9^{\log_3 x} \Longrightarrow x^{\log_3 9} = x^2$

$x > 0$



ب) $y = \log x^2 = 2\log x$

$x^2 > 0 \rightarrow D = R$