نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

$(a+c)b = ?$

$b+c = \frac{-3}{2}$: I $\quad y = 1 - \log_c(ax-b)$

$\begin{cases} x=0 \\ y=2 \end{cases} \Rightarrow \cancel{2} = 1 - \log_c^{-b} \to \log_c^{-b} = -1 \to \overset{\frac{1}{c}}{c^{-1}} = -b \to b = -\frac{1}{c} \xrightarrow{I} \frac{-1}{c} + c = \frac{-3}{2} \xrightarrow{\times 2c} -2 + 2c^2 = -3c$

$2c^2 + 3c - 2 = 0 \Rightarrow (c+2)(c-1) = 0$, $\begin{cases} c^2+3c-2=0 \\ c = -\frac{4}{2} = -2 \\ c = \frac{1}{2} \end{cases}$

1) $c = -2 \to b = \frac{1}{2} \xrightarrow{II} \frac{-3}{2} = \frac{-3}{2}a - \frac{1}{2} \to a = 1 \longrightarrow (a+c)b = (1-2)\frac{1}{2} = -\frac{1}{2}$ جواب

2) $c = \frac{1}{2} \to b = -2 \xrightarrow{II} \frac{1}{2} = \frac{-3}{2}a + \frac{4}{2} \to -\frac{3}{2} = \frac{-3}{2}a \to a = 1 \longrightarrow (a+c)b = (1+\frac{1}{2})(-2) = -3$ جواب

$\begin{cases} x = \frac{-3}{2} \\ y = 0 \end{cases} \Rightarrow \cancel{0} = \cancel{1} + \log_c \to c = -\frac{3}{2}a - b$: II

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$f(-1) = ?$

$f(x) = 1 + c \times 3^{ax+bx} \xrightarrow{x=-1} f(1) = 1 + (-1) \times 3^{\frac{(-1)+(-1)}{3-2}} = 1 - \frac{1}{9} = \frac{8}{9}$ ⇒ جواب

$\begin{cases} x=1 \\ y=0 \end{cases} \Rightarrow 0 = 1 + c \times 3^{a+b} \xrightarrow[a=-1]{c=-1} 0 = 1 + (-1) \times 3^{(-1)+b}$

$\cancel{-1} = \cancel{1} 3^{-1+b} \Rightarrow -1 + b = 0 \to b = 1$

$\begin{cases} x = 0 \\ y = \frac{2}{3} \end{cases} \Rightarrow \frac{2}{3} = \overset{\frac{3}{3}}{1} + c \times 3^a \to c \times 3^a = -1 \times 3^{-1}$

$\to c = -1$, $3^a = 3^{-1} \Rightarrow a = -1$

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$\frac{a}{b} = ?$

$y = c + \log_5(ax+b)$

$\begin{cases} x=0 \\ y=2 \end{cases} \Rightarrow 2 = c + \log_5^b \to 2 - c = \log_5^b \Rightarrow 5^{2-c} = b = 5^2 \times 5^{-c} = b \to 5^{-c} = \frac{b}{25}$

$\begin{cases} x = 2.4 \\ y = 0 \end{cases} \Rightarrow 0 = c + \log_5^{2.4a+b} \to -c = \log_5^{2.4a+b} \Rightarrow 5^{-c} = 2.4a + b \Rightarrow \frac{b}{25} = 2.4a + b \to \frac{2.4b}{25}$

$-\frac{2.4}{25}a = \frac{2.4b}{25} \to \frac{a}{b} = \frac{-2}{5}$

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$f(x) = \log_4(|x^2-2| - x)$

$(x-2)(x+1) > 0$ ($x=2$, $x=-1$)

$|x^2-2| - x > 0 \to$ (1) $x^2 - 2 - x > 0 \Rightarrow x^2 - x - 2 > 0$; I : $(-\infty, -1) \cup (2, +\infty)$

(2) $-x^2 + 2 - x > 0 \Rightarrow -x^2 - x + 2 > 0$; $-1 - 1 + 2 = 0 \to a + b + c = 0 \to x: \begin{cases} 1 \\ = \frac{c}{a} = -2 \end{cases}$; II : $(-2, 1)$

$D_f = (-2, -1)$ جواب

$D_f : I \cap II :$ $-2 \quad -1 \quad 1 \quad 2$

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$rb - a = ?$

$f(x) = 2 + 2^{b-ax}$, $g(x) = -x^2 - 3x + 8$ $\quad f^{-1}(10) = -1 \to f(-1) = 10$

$x=1 \to f(1) = g(1) \to \cancel{2} + 2^{b-a} = -1 - 3 + 8 \Rightarrow 2^{b-a} = 2 \to b - a = 1$

$x = -1$, $f(-1) = \cancel{2} + 2^{b+a} = \cancel{10} \to 2^{b+a} = 2^3 \Rightarrow b + a = 3 \to 2b = 4 \to b = 2, a = 1$

$2b - a = 2(2) - (1) = 4 - 1 = 3$ → جواب

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$f(x) = -2 + \left(\frac{1}{2}\right)^{Ax+B}$ $\qquad y = x^2 - x$

$x = 1 \Rightarrow -2 + \left(\frac{1}{2}\right)^{A+B} = \overbrace{(1)^2 - 1}^{0} \Rightarrow \left(\frac{1}{2}\right)^{A+B} = 2 = \left(\frac{1}{2}\right)^{-1} \Rightarrow A + B = -1 \xrightarrow{\times 2} 2A + 2B = -2$

$x = 2 \Rightarrow -2 + \left(\frac{1}{2}\right)^{2A+B} = \overbrace{4 - 2}^{2} \Rightarrow \left(\frac{1}{2}\right)^{2A+B} = 4 = \left(\frac{1}{2}\right)^{-2} \Rightarrow 2A + B = -2$

$\Rightarrow \alpha A + 2B = \alpha A + B \Rightarrow B = 0 \Rightarrow A = -1$

$x = 3$

$\xrightarrow{} f(x) = -2 + \left(\frac{1}{2}\right)^{-x} \Rightarrow f(3) = -2 + \left(\frac{1}{2}\right)^{-3} = 6$ جواب

$P(t) = A_0 \times a^{\frac{t}{m}}$ $\Rightarrow P(t) = \frac{1}{6} A_0 = A_0 \times \left(\frac{8}{9}\right)^{\frac{t}{1}} \rightarrow \frac{1}{6} = \left(\frac{8}{9}\right)^t$

چند برابر شدن مقدار اولیه — بلافاصله — هر واحد زمانی

$\xrightarrow{\text{لگاریتم مبنای ۵}} \log_5 \left(\frac{8}{9}\right)^t = \log_5 \frac{1}{6} \Rightarrow t \log_5 \left(\frac{8}{9}\right) = -\log_5 6 \Rightarrow t\left(-\frac{\cancel{5}}{\cancel{24}}\right) = -\frac{\cancel{5}}{\cancel{24}}$

$a = 1 - \frac{1}{9} = \frac{8}{9}$

$\log_3 5 \simeq 1.4 = \frac{7}{5} \rightarrow \log_5 3 = \frac{5}{7}$

$\log_2 5 \simeq 2.3 = \frac{12}{5} \rightarrow \log_5 2 = \frac{5}{12}$

$\Rightarrow \log_5 6 = \log_5 3 + \log_5 2 = \frac{5}{7} + \frac{5}{12} = \frac{60 + 35}{84} = \frac{95}{84}$

$\Rightarrow \log_5 \frac{8}{9} = \log_5 8 - \log_5 9 = 3\log_5 2 - 2\log_5 3 = \frac{5}{4} - \frac{10}{7} = \frac{35 - 40}{28} = -\frac{5}{28}$

$\Rightarrow t = \frac{19}{3} \times 60 = 380$ دقیقه $= 6$ ساعت

جواب

$P(t) = A_0 \times a^{\frac{t}{m}} \rightarrow \frac{1}{7} A_0 = A_0 \times \left(\frac{7}{8}\right)^{\frac{t}{1}} \rightarrow \frac{1}{7} = \left(\frac{7}{8}\right)^{\frac{t}{7}} \xrightarrow{\log_3} \log_3 \left(\frac{7}{8}\right)^{\frac{t}{7}} = \log_3 7^{-1}$

$a = 1 - \frac{12.5}{100} = \frac{87.5}{100} = \frac{87.5 \times 7}{100 \times 8} = \frac{7}{8}$

$\rightarrow \frac{t}{7} \log_3 \frac{7}{8} = -1 \log_3 7 \rightarrow \frac{t}{7}\left(\log_3 7 - \log_3 8\right) = -\log_3 7$

$\log_3 7 = \frac{6}{1.5} \rightarrow \log_3 7 = \frac{5}{3}$

$\log_3 8 = \frac{8}{5} \rightarrow \log_3 2 = \frac{5}{8}$

$\frac{t}{7}\left(\frac{5}{3} - \frac{15}{8}\right) = -\frac{5}{3} \rightarrow \frac{t}{7}\left(\frac{-40 - 45}{24}\right) = -\frac{5}{3}$

$\rightarrow t = 56$ جواب

$P(t) = A_0 \times a^{\frac{t}{m}} \longrightarrow \frac{1}{3} A_0 = A_0 \times \left(\frac{96}{100}\right)^{\frac{t}{1}} \rightarrow \frac{1}{3} = \left(\frac{96}{100}\right)^t \xrightarrow{\log_{10}} \log \frac{1}{3} = \log \frac{96}{100}\, t$

تفاوت حجم هر روز $\frac{96}{100}$ برابر می‌شود

حجم اولیه $- 100$ لیتر — حجم اولیه $- 100$ لیتر

$= \frac{96}{100} = a$

$\rightarrow -\log 3 = t(\log 96 - \log 100)$

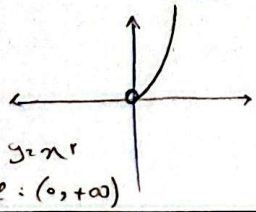
$96 = 2^5 \times 3$

$-\frac{48}{100} = t\left(5\log 2 + \log 3 - 2\right) \rightarrow -\frac{48}{100} = t\left(-\frac{7}{100}\right) \rightarrow t = 24$ روز

الف) $y = 9^{\log_3 x} > 0$

$9^{\log_3 x} = x^{\log_3 9} = x^2$

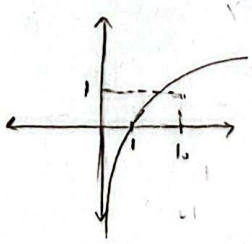
$x > 0$



$y = x^2$

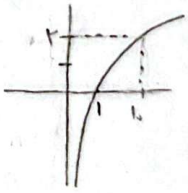
$D_f : (0, +\infty)$

ب) $y = \log x^2 = 2\log x$



$y = \log x$

$\xrightarrow{\times 2}$



$y = 2\log x$