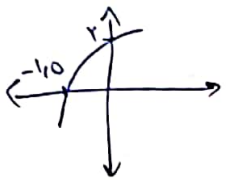


سوال ۱:

$y = 1 - \log_c^{(ax-b)}$ ، $b+c = -\frac{3}{2}$ ، $(a+c)b = ?$

$(0,2) \to 1 - \log_c^{-b} = 2 \to \log_c^{-b} = 1 \to -b = \frac{1}{c}$ $\Big\} \to b = \frac{-1}{c}$

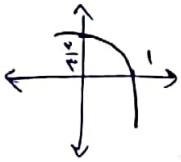
$(-\frac{3}{2}, 0) \to 1 - \log_c^{-\frac{3}{2}a-b} = 0 \to -\frac{3}{2}a - b = c \to \frac{-3}{2}a = b + c \to a = 1$ ✓

$b + c = \frac{-3}{2} \to -\frac{1}{c} + c = -\frac{3}{2} \to \frac{1-c^2}{c} = \frac{3}{2} \to 2c^2 + 3c - 2 = 0$

$\Delta = 9 + 16 = 25$ ، $c = \frac{-3 \pm 5}{4} \to c = \frac{1}{2}$ ✓ ، $c = -2$ ✗

$-b + \frac{1}{c} = 2 \to -b + 2 = 2 \to b = 0$

$(a+c)b = (1 + \frac{1}{2}) \times (-2) = -3$



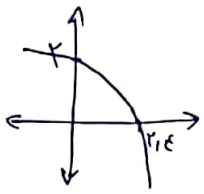
سوال ۲:

$f(x) = 1 + c \times 3^{ax+b}$ ، $f(-1) = ?$

$(1,0) \to 1 + c \times 3^{a+b} = 0 \to c \times 3^{a+b} = -1$

$(0, \frac{2}{3}) \to 1 + c \times 3^{a} = \frac{2}{3} \to c \times 3^{a} = \frac{-1}{3}$ $\Big\} \div \to 3^b = 3 \to b = 1$ ✓

$f(x) = 1 + \underbrace{c \times 3^{a}}_{\frac{-1}{3}} \times 3^{x} \xrightarrow{f(-1)} 1 - \frac{1}{3} \times 3^{-1} = 1 - \frac{1}{3} \times \frac{1}{3} = \frac{8}{9}$



سوال ۳:

$y = c + \log_5^{(ax+b)}$ ، $\frac{a}{b} = ?$

$(0,2) \to c + \log_5^{b} = 2$

$(2,4,0) \to (c + \log_5^{2,4a+b} = 0)^{-1}$ $\Big\} \div \to \log_5^{\frac{b}{2,4a+b}} = 2 \to \frac{b}{2,4a+b} = 25$ ✓

$b = 70a + 252b \to 24b = -20a$

$\frac{a}{b} = \frac{-24}{20} = \frac{-6}{5}$

سوال ۴:

$f(x) = \log_4^{(|x^2-2|-x)}$ $\to |x^2-2| - x > 0$

① $x^2 - 2 \geq 0 \to x^2 \geq 2 \to x \geq \sqrt{2}$ یا $x \leq -\sqrt{2}$ ، $x^2 - 2 - x > 0 \to$ (ریشه‌ها: 2 و -1) $\to x < -1$ یا $x > 2$

$\to x \leq -\sqrt{2}$ یا $x > 2$ ✓

② $x^2 - 2 < 0 \to x^2 < 2 \to -\sqrt{2} < x < \sqrt{2}$ ، $-x^2 + x + 2 > 0 \to x^2 - x - 2 < 0 \to (x-2)(x+1) < 0 \to -1 < x < 2$

$\to -1 < x < \sqrt{2}$ ✓

① ∪ ② $\to D_f = (-\infty, -1) \cup (2, +\infty)$

سوال ۵:

$f(x) = 2 + 2^{b-ax}$ ، $g(x) = -x^2 - 3x + 4$ $\Big\} \to (1,0) \in f^{-1}(1) = 1$ ، $2b - a = ?$

$2 + 2^{b-a} = -1 - 3 + 8 \to 2^{b-a} = 2 \to b - a = 1$

$f^{-1}(1) = 1 \xrightarrow{\text{وارون}} (1,0)$ نقطه $\to 2 + 2^{a+b} = 10 \to 2^{a+b} = 8 = 2^3 \to a + b = 3$

$b - a = 1$

$a + b = 3$

$2b = 4 \to b = 2$

$a = 1$

$2b - a = 3 \quad (2) - 1 = 3$

سوال ۶ -

$f(x) = -2 + (\frac{1}{2})^{Ax+B}$ $\Big\}$ $(1,\square) \to (2,\square)$ $\Rightarrow f(3) = ?$
$y = x^2 - x$

$x^2 - x \xrightarrow{x=1} 1-1=0 \to (1,0)$
$\xrightarrow{x=2} 4-2=2 \to (2,2)$ $\Big\}$ این نقاط مسیر تابع $f(x)$ است

$(1,0) \to -2 + (\frac{1}{2})^{A+B} = 0 \to (\frac{1}{2})^{A+B} = 2 \to A+B = -1$ (۱)
$(2,2) \to -2 + (\frac{1}{2})^{2A+B} = 2 \to (\frac{1}{2})^{2A+B} = 4 \to 2A+B = -2$ (۲)
$\Rightarrow A = -1,\ B = 0$

$f(x) = -2 + (\frac{1}{2})^{-x} \xrightarrow{f(3)} -2 + (\frac{1}{2})^{-3} = -2 + 8 = 6$ (ج)

سوال ۷: (۱۷۵)

$m = m_0 \times (\frac{5}{9})^t$ (جرم اولیه) $\to \frac{1}{4} m_0 = m_0 (\frac{5}{9})^t \Rightarrow \frac{1}{4} = (\frac{5}{9})^t \to \log_{\frac{5}{9}} \frac{1}{4} = t$ (زمان)

$\log_{\frac{5}{9}} \frac{1}{4} = \frac{\log_5 1 - \log_5 4}{\log_5 9 - \log_5 5} = \frac{-\log_5 4}{2\log_5 3 - 1} = -(\log_5 4 + \log_5 3) = -(2\log_5 2 + \log_5 3)$

$\log_5 2 = \frac{1}{\log_2 5} = \frac{1}{2.3} = \frac{1}{6}$

$\log_5 3 = \frac{1}{\log_3 5} = \frac{1}{1.46} = \frac{1}{\frac{7}{5}} = \frac{5}{7}$

$-(\frac{2}{6} + \frac{5}{7}) = -(\frac{1}{3} + \frac{5}{7}) = -\frac{22}{21}$

$\frac{22}{21} \times 90 = 22 \times 30 = 660$ min

سوال ۸: (۱۹/۴)

$m = m_0 a (\frac{1}{8})^t \to \frac{1}{8} m_0 = m_0 (\frac{1}{8})^t \to \frac{1}{8} = (\frac{1}{8})^t \to \log_{\frac{1}{2}} \frac{1}{8} = t$

$\log_{\frac{1}{2}} \frac{1}{8} = \frac{\log_2 1 - \log_2 8}{\log_2 1 - \log_2 2} = \frac{-3}{-1} = 3 = t$

$\frac{1}{10} = \frac{1}{8} a \to a = \frac{8}{10} = \frac{4}{5}$

$(\frac{4}{5})^{10} = (\frac{2}{5})^5 = \frac{32}{3125}$

$\frac{32}{3125} \times 100 = \frac{32}{125}$ درصد $\to 1 - \frac{32}{125} = \frac{93}{125}$ (ج)

سوال ۹:

$a = 90 (\frac{99}{100})^n \to \frac{1}{2} a_0 = a_0 (\frac{99}{100})^n \to \frac{1}{2} = (\frac{99}{100})^n \to \log \frac{1}{2} = \log (\frac{99}{100})^n$

$-\log 2 = n(\log 99 - \log 100) = n(\log 3^2 + \log 11 - \log 10^2) = n(2\log 3 + \log 11 - 2)$

$n = \frac{-0.3}{0.0086} \approx 34.8$ (ج)

سوال ۱۰:

$y = 9 \log_3 x \to x = 3^{\frac{y}{9}}$ (است) $\to y = 2\log x^2$

$x > 0$