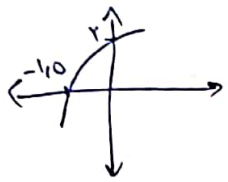
سوال ۱:



$y = 1 - \log_c^{(ax-b)}$ ، $b+c = -\frac{3}{2}$ ، $(a+c)b = ?$

$(0,2) \to 1 - \log_c^{-b} = 2 \to \log_c^{-b} = 1 \to -b = \frac{1}{c}$ $\Big\} \to b = \frac{-1}{c}$

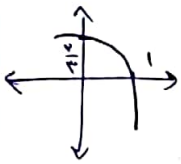
$(-\frac{3}{2}, 0) \to 1 - \log_c^{-\frac{3}{2}a-b} = 0 \to -\frac{3}{2}a - b = c \to \frac{-3}{2}a = \overbrace{b+c}^{-\frac{3}{2}} \Rightarrow \boxed{a=1}$

$b+c = \frac{-3}{2} \to -\frac{1}{c} + c = -\frac{3}{2} \to \frac{1-c^2}{c} = \frac{3}{2} \to 2c^2 + 3c - 2 = 0$ $\to \boxed{c = \frac{1}{2}}$ ، $c = -2$ غ ق ق

$\boxed{b = -2}$ ، $-b + \frac{1}{c} = 2 + 2c$

$(a+c)b = (1+\frac{1}{2}) \times (-2) = \boxed{-3}$

سوال ۲:



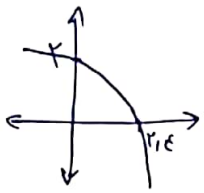
$f(x) = 1 + c \times 3^{ax+b}$ ، $f(-1) = ?$

$(1,0) \to 1 + c \times 3^{a+b} = 0 \to c \times 3^{a+b} = -1$

$(0, \frac{2}{3}) \to 1 + c \times 3^{a} = \frac{2}{3} \to c \times 3^{a} = \frac{-1}{3}$ $\Big\} \div \to 3^b = 3 \to \boxed{b=1}$

$f(x) = 1 + \underbrace{c \times 3^{a}}_{\frac{-1}{3}} \times 3^{x} = 1 - \frac{1}{3} \times 3^{x} \xrightarrow{f(-1)} 1 - \frac{1}{3} \times \frac{1}{3} = \boxed{\frac{8}{9}}$

سوال ۳:



$y = c + \log_5^{(ax+b)}$ ، $\frac{a}{b} = ?$

$(0,2) \to c + \log_5^{b} = 2$

$(2,4,0) \to (c + \log_5^{2,4a+b} = 0)^{-1}$ $\Big\} + \to \log_5^{\frac{b}{2,4a+b}} = 2 \to \frac{b}{2,4a+b} = 25$

$b = 70a + 252b \to 24b = -20a$

$\boxed{\frac{a}{b} = \frac{-24}{20} = \frac{-6}{5}}$

سوال ۴:

$f(x) = \log_{\frac{1}{2}}^{(|x^2-2|-x)}$ $\to |x^2-2| - x > 0$

① $x^2 - 2 \geq 0 \to x^2 \geq 2 \to x \geq \sqrt{2}$ ، $x \leq -\sqrt{2} \to x^2 - x - 2 > 0 \to$ (−1, 2) $\to x < -1$ ، $x > 2$

$\xrightarrow{\cap}$ ① $x > 2$ ، $x \leq -\sqrt{2}$

② $x^2 - 2 < 0 \to -\sqrt{2} < x^2 < \sqrt{2} \to -x^2 + 2 > x^2 > x - 2 \to x^2 + x - 2 < 0 \to (x+2)(x-1) < 0 \to -2 < x < 1$ $\xrightarrow{\cup}$ ② $-\sqrt{2} < x < 1$

$\xrightarrow{\cup}$ ① ∪ ② $\to D_f = (-\infty, 1) \cup (2, +\infty)$

سوال ۵:

$f(x) = 2 + 2^{b-ax}$

$g(x) = -x^2 - 3x + 4$ $\Big\} \to (1,0) \in f^{-1}(1) \to 1 = f^{-1}(1)$ ، $2b - a = ?$

$2 + 2^{b-a} = -1 - 3 + 8 \to 2^{b-a} = 2 \to b - a = 1$

$f^{-1}(1) = 1 \xrightarrow{\text{متقارن}} (1,0)$ نقطه $\to 2 + 2^{a+b} = 1 \to 2^{a+b} = -1$ غ ق ق $\to a + b = 3$

$b - a = 1$

$a + b = 3$

$2b = 4 \to b = 2$

$a = 1$

$\boxed{2b - a = 3 \ (2) - 1 = 5}$

سوال ۶-

$f(x) = -2 + (\frac{1}{2})^{Ax+B}$, $y = x^2 - x$: $(1,\square) \to (2,\square)$ ؟ $f(3) = ?$ $f \circ g$

$x^2 - x \xrightarrow{x=1} 1-1=0 \to (1,0)$; $\xrightarrow{x=2} 4-2=2 \to (2,2)$ ← این نقاط بر $f(x)$ مماس می‌گذرند

$f(x) = -2 + (\frac{1}{2})^{-x} \xrightarrow{f(3)} -2 + (\frac{1}{2})^{-3} = -2 + 8 = 6$

$(1,0) \to -2 + (\frac{1}{2})^{A+B} = 0 \to (\frac{1}{2})^{A+B} = 2 \to A+B = -1$ ×(-1)

$(2,2) \to -2 + (\frac{1}{2})^{2A+B} = 2 \to (\frac{1}{2})^{2A+B} = 4 \to 2A+B = -2$

$\to A = -1$, $B = 0$

سوال ۷:

$m = m_0 \times (\frac{5}{9})^t$ (جرم اولیه) $\to \frac{1}{4} m_0 = m_0 (\frac{5}{9})^t \Rightarrow \frac{1}{4} = (\frac{5}{9})^t \to \log_{\frac{5}{9}} \frac{1}{4} = t$ (زمان نیمه عمر)

$\log_{\frac{5}{9}} \frac{1}{4} = \frac{\log_5 1 - \log_5 4}{\log_5 9 - \log_5 5} = \frac{0 - 2\log_5 2}{2\log_5 3 - 1} = \frac{-2\log_5 2}{2\log_5 3 - \log_5 5} = -(\log_5 4 + \log_5 9)$

$= -(\frac{1}{\log_4 5} + \frac{1}{\log_9 5}) = -(\frac{1}{\frac{1}{2}\log_2 5} + \frac{1}{\frac{1}{2}\log_3 5}) = -(\frac{2}{\log_2 5} + \frac{2}{\log_3 5}) = -2(\log_5 2 + \log_5 3)$

$= -2\log_5 6 \to t = 2$ (سال)

$\log_5 6 = \frac{1}{\log_6 5} = \frac{1}{\log_6 \frac{10}{2}} = \frac{1}{1 - \log_6 2} = \frac{1}{1 - \frac{1}{\log_2 6}} = \frac{1}{1 - \frac{1}{1 + \log_2 3}} = \frac{1}{\frac{\log_2 3}{1 + \log_2 3}} = \frac{1 + \log_2 3}{\log_2 3}$

$\log_5 2 = \frac{1}{\log_2 5} = \frac{1}{\log_2 \frac{10}{2}} = \frac{1}{1 - \log_2 2} = \frac{1}{1 - \frac{1}{\log_5 2}} = \frac{1}{1 - \frac{1}{1 + \log_5 3}} = \frac{1}{\frac{\log_5 3}{1 + \log_5 3}} = \frac{1 + \log_5 3}{\log_5 3}$

سوال ۸:

$m = m_0 a^t \to \frac{1}{8} m_0 = m_0 (\frac{1}{2})^t \to \frac{1}{8} = (\frac{1}{2})^t \to \log_{\frac{1}{2}} \frac{1}{8} = t$

$\log_{\frac{1}{2}} \frac{1}{8} = \frac{\log_2 1 - \log_2 8}{\log_2 1 - \log_2 2} = \frac{0 - 3\log_2 2}{0 - 1} = 3$ سال

$\frac{1}{8} = \frac{1}{100} \times 12.5$

$\log_{100} 12.5 = \log_{10^2} 12.5 = \frac{1}{2}\log_{10} \frac{125}{10} = \frac{1}{2}(\log_{10} 125 - \log_{10} 10) = \frac{1}{2}(3\log_{10} 5 - 1)$

$= \frac{1}{2}(3\log_{10} \frac{10}{2} - 1) = \frac{1}{2}(3 - 3\log_{10} 2 - 1) = \frac{1}{2}(2 - 3\log_{10} 2)$

$= \frac{1}{2}(2 - 3 \times 0.301) = \frac{1}{2}(2 - 0.903) = \frac{1.097}{2} = 0.5485$

$0.5485 \times 100 = 54.85$ دقیقه

سوال ۹:

$a = 9.0 \times (\frac{96}{100})^n \to \frac{1}{3} a_0 = a_0 (\frac{96}{100})^n \to \frac{1}{3} = (\frac{96}{100})^n \to \log_{\frac{96}{100}} \frac{1}{3} = n$

$\log_{\frac{96}{100}} \frac{1}{3} = \frac{\log_{10} 1 - \log_{10} 3}{\log_{10} 96 - \log_{10} 100} = \frac{0 - \log_{10} 3}{\log_{10} 3 + 5\log_{10} 2 + 2\log_{10} 2 - 2} = \frac{-\log_{10} 3}{-2 + 3\log_{10} 3 + 7\log_{10} 2}$

$= \frac{-0.477}{-2 + 1.431 + 2.107} = \frac{-0.477}{1.538} = -0.31$

$n = 0.31 \times 365 = 113.15$ روز

سوال ۱۰:

$y = \log_3 x^2 \to x > 0$

$y = 2\log_3 x$ (است) $\to x = 3^{\frac{1}{2}y}$