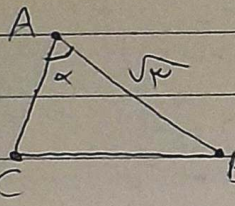
۱۱

1



(۲۰)

(۹)

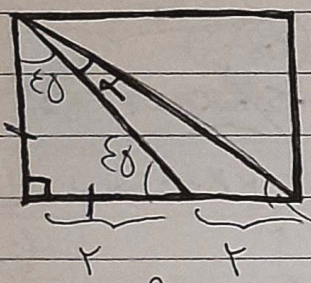
$\frac{۱}{۲}\times ۴\times\sqrt{۳}\sin\alpha = \frac{۹}{۲}\sqrt{۳}$

جمع زوایای داخلی مثلث مثلث کمتر از ۱۸۰ است $\alpha<۱۸۰$

$\sin\alpha = \frac{\sqrt{۳}}{۲} \rightarrow \alpha = ۶۰^\circ$ یا $\alpha = ۱۲۰^\circ$

$\frac{۱۲۰}{۹۰} = \frac{۴}{۳}$

2



$\cot\alpha = ?$

$\tan(۴۵-\alpha) = \frac{\tan۴۵ - \tan\alpha}{۱+\tan۴۵\times\tan\alpha} = \frac{۱}{۲}$

$\frac{۱-\tan\alpha}{۱+\tan\alpha} = \frac{۱}{۲}$ ترکیب دخرج

$\frac{۱-\tan\alpha}{۲} = \frac{۱}{۳}$

$۳-۳\tan\alpha = ۲$

$۱-۳\tan\alpha = ۲$

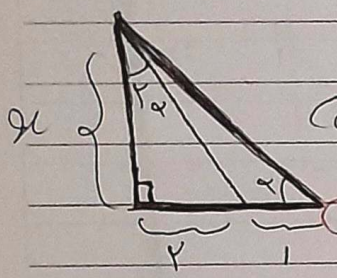
$-۳\tan\alpha = ۱$

$\tan\alpha = -\frac{۱}{۳}$

$\cot\alpha = -۳$

(۹)

3



$\cot\alpha = ?$

$\cot\alpha = \frac{۳}{۴x} \rightarrow \tan\alpha = \frac{۴x}{۳}$

$\cot۲\alpha = \frac{۹-۴x^۲}{۶x} \rightarrow \tan۲\alpha = \frac{۶x}{۹-۴x^۲}$

$\tan(\alpha+۲\alpha) = \frac{\tan\alpha + \tan۲\alpha}{۱-\tan\alpha\tan۲\alpha}$

$\frac{۱}{x} = \frac{\frac{۴x}{۳} + \frac{۶x}{۹-۴x^۲}}{۱-\frac{۴x}{۳}\times\frac{۶x}{۹-۴x^۲}}$

$\frac{۱}{x} = \frac{\frac{۴x(۹-۴x^۲)+۱۸x}{۳(۹-۴x^۲)}}{\frac{۳(۹-۴x^۲)-۲۴x^۲}{۳(۹-۴x^۲)}}$

$\frac{۱}{x} = \frac{۳۶x-۱۶x^۳+۱۸x}{۲۷-۱۲x^۲-۲۴x^۲}$

$\frac{۱}{x} = \frac{۵۴x-۱۶x^۳}{۲۷-۳۶x^۲}$

$۲۷-۳۶x^۲ = ۵۴x^۲-۱۶x^۴$

$۱۶x^۴-۹۰x^۲+۲۷ = ۰$

$x^۲ = \frac{۹۰\pm\sqrt{۶۰۴}}{۳۲} = \frac{۹۰\pm۲۲}{۳۲}$

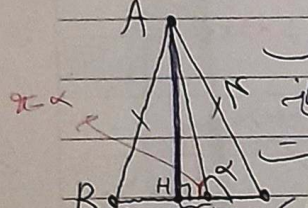
$x^۲ = ۳ \rightarrow x = \sqrt{۳}$

$x^۲ = \frac{۹}{۴} \rightarrow x = \frac{۳}{۲}$

$\cot\alpha = ۲$

(۱۰)

4



ارتفاع وارد بر وتر در مثلث قائم‌الزاویه برابر است با حاصل ضرب دو ضلع قائم تقسیم بر وتر

$AH = \frac{۱\times۲}{۳}$

$AH = \sqrt{۴-۹}$

$AH = \sqrt{۷}$

$\tan\alpha = \frac{\sqrt{۷}}{۲}$

$\tan(x-\alpha) = -\tan\alpha$

$\tan(x-\alpha) = \sqrt{۷}$

$\tan\alpha = -\frac{\sqrt{۷}}{۲}$

(۹)

5

$۲\sin^۲x + \cos^۲x = \frac{۴}{۳}$

$۲(۱-\cos^۲x)+\cos^۲x = \frac{۴}{۳} \rightarrow ۲-۲\cos^۲x+\cos^۲x = \frac{۴}{۳}$

$۲-\cos^۲x = \frac{۴}{۳}$

$\frac{۲}{۳} = \cos^۲x$

$\tan^۲x = \frac{۱}{\cos^۲x} - ۱ \rightarrow \tan^۲x = \frac{۳}{۲} - ۱$

$\frac{۱}{۲}$

$\tan^۲x = ?$

(۹)

6

$$\frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha}$$

$4(1-\sin^2\alpha)$, $4(1-\cos^2\alpha)$

$$\frac{\sin^4\alpha - 4\sin^2\alpha + 4}{1+\cos^2\alpha} = \frac{\cos^4\alpha - 4\cos^2\alpha + 4}{1+\sin^2\alpha} = \frac{(\sin^2\alpha - 2)^2}{1+\cos^2\alpha} - \frac{(\cos^2\alpha - 2)^2}{1+\sin^2\alpha}$$

$1-\cos^2$, $1-\sin^2$

$$\frac{(-\cos^2\alpha - 1)^2}{1+\cos^2\alpha} - \frac{(-\sin^2\alpha - 1)^2}{1+\sin^2\alpha}$$

$$\cos^2\alpha + 1 - \sin^2\alpha - 1$$

$$\cos 2\alpha$$

7

$\tan\alpha = \frac{4}{3}$

$\pi < \alpha < \frac{3\pi}{2}$

$$1+\tan^2\alpha = \frac{1}{\cos^2\alpha} \to \cos^2\alpha = \frac{9}{25} \xrightarrow{\frac{3}{4}\cos^2\alpha} \frac{27}{100}$$

$1+\frac{16}{9}$, $\frac{25}{9}$

$$\frac{3}{4}\,\frac{1}{\tan\alpha}(-\sin^2\alpha+1) \to \frac{3}{4}\cos^2\alpha$$

$-\tan\beta = \tan(-\beta)$

$$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right) - \tan\left(\alpha-\frac{3\pi}{2}\right)$$

$$\frac{\tan\alpha}{\tan\alpha}\left(\cos\alpha \times -\sin\alpha + \cot\alpha\right)$$

$\sin\left(4\pi + \left(\frac{\pi}{2}+\alpha\right)\right)$; $\cos\left(4\pi - \left(\frac{\pi}{2}+\alpha\right)\right)$; $4\pi - \frac{\pi}{2} - \alpha$; $\tan\left(\frac{3\pi}{2}-\alpha\right) \to \cot\alpha$

$\sin(4\pi+\theta) = \sin\theta$; $\sin\left(\frac{\pi}{2}+\alpha\right) \to \cos\alpha$

$\cos(4\pi-\theta) = \cos(\theta)$; $\cos\left(\frac{\pi}{2}+\alpha\right) \to \sin\left(\frac{\pi}{2}+\alpha\right) \to -\sin\alpha$

8

$x = \frac{\pi}{12}$

$$3\cos 4x + \sqrt{2}\sin x - \sqrt{2}\cos x \to \frac{3}{2} - 1 = \frac{1}{2}$$

$3\cos\frac{\pi}{3}$, $\frac{1}{2}$, $\frac{3}{2}$

$\sqrt{2}(\sin x - \cos x)$

$\sqrt{2}\times\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)$ } $2\sin\left(\frac{\pi}{12}-\frac{\pi}{4}\right)$ ($-\frac{\pi}{6}$) } $-2\sin\frac{\pi}{6}$ ($\frac{1}{2}$) → -1

9

$\frac{1}{4} = \tan\left(\frac{\alpha}{2}\right)$ طبق فرض سوال ✗ ← $0 > \tan\left(\frac{\alpha}{2}\right)$ ← $\frac{3\pi}{4} > \frac{\alpha}{2} > \frac{\pi}{2}$ ← $\frac{3\pi}{2} > \alpha > \pi$ ←

$$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \frac{\frac{17\times 8 - 15\times 8}{17\times 15}}{\frac{-7}{17}} \to \frac{-16}{105}$$

$\frac{\pi}{2} > \alpha > 0$ ✓ ← $\tan\alpha > 0$

$$\tan\left(\frac{\alpha}{2}\right) = \frac{1}{4} \to \tan\left(\frac{\alpha}{2}+\frac{\alpha}{2}\right) = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} \to \tan\alpha = \frac{2\times\frac{1}{4}}{1-\frac{1}{16}} = \frac{8}{15}$$

10

$$0 < \frac{\cot\alpha}{\sin\alpha} \to \frac{\frac{\cos\alpha}{\sin\alpha}}{\sin\alpha} > 0 \;\Big\}\; \frac{\cos\alpha}{\sin^2\alpha} > 0 \xrightarrow{\sin^2\alpha > 0} \cos\alpha > 0$$

$0 \neq \sin\alpha$ نتیجه

$2\sin\alpha < \sin 2\alpha$ ↳ $2\sin\alpha < 2\sin\alpha\cos\alpha \to 1 < \cos\alpha \to$ غیر ممکن است

اشتباه کردن!

به فرض اینکه $\sin\alpha > 0$ حساب کردیم ↳ $\sin\alpha < 0$, $\cos\alpha > 0$ } ناحیه چهارم

Parsian