نام و نام خانوادگی: آیین عظیمی — تکلیف: ۲۷ — کلاس: پنجم یازدهم ریاضی

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$S = \frac{1}{2} ab\sin\alpha \rightarrow \frac{1}{2} \times 6 \times \sqrt{3} \times \sin\alpha = \frac{9}{2}$

$\sin\alpha = \frac{\sqrt{3}}{2}$ ، $\alpha = 60$ ، $\alpha = 120$

$\frac{120}{60} = (2)$ برابر

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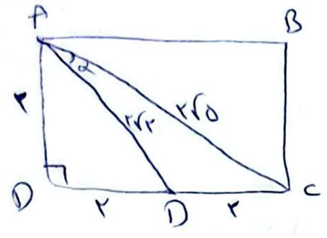
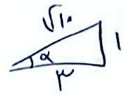


$S_{\overset{\triangle}{ADC}} = \frac{AD \cdot DC}{2} = 2 = \frac{1}{2} AC \cdot AD \cdot \sin\alpha$

$AC = \sqrt{16+4} = 2\sqrt{5}$ ، $AD = 2\sqrt{2}$

$\Rightarrow \frac{1}{2} \times 2\sqrt{2} \times 2\sqrt{5} \times \sin\alpha = 2$

$\sin\alpha = \frac{1}{\sqrt{10}}$



$\cot\alpha = (3)$

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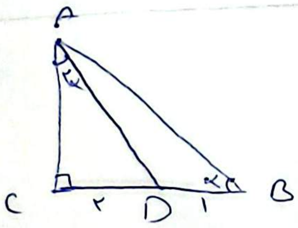

$\tan 2\alpha \; (\overset{\triangle}{ACD}) = \frac{2}{AC} = \frac{2\tan\alpha}{1-\tan^2\alpha}$

$\tan\alpha \; (\overset{\triangle}{ABC}) = \frac{AC}{3}$

$\Rightarrow \frac{2}{AC} = \frac{\frac{2}{3}AC}{1-\frac{AC^2}{9}} \rightarrow 2 - \frac{2}{9}AC^2 = \frac{2}{3}AC^2$

$\frac{8}{9}AC^2 = 2$

$AC^2 = 2 \times \frac{9}{8} = \frac{9}{4}$

$AC = \frac{3}{2}$

$\cot\alpha \; (\overset{\triangle}{ABC}) = \frac{BC}{AC} = \frac{3}{\frac{3}{2}} = (2)$

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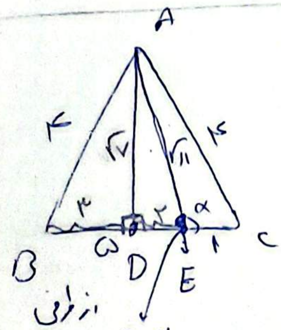

ارتفاع AD عمود بر BC $\rightarrow AD = \sqrt{16-9} = \sqrt{7}$

$AE = \sqrt{7+4} = \sqrt{11} \rightarrow S_{\overset{\triangle}{AEC}} = \frac{\sqrt{7} \times 1}{2} = \frac{1}{2} AE \cdot EC \cdot \sin\alpha$

$\frac{\sqrt{7}}{2} = \frac{1}{2} \times \sqrt{11} \times 1 \times \sin\alpha$

$\sin\alpha = \sqrt{\frac{7}{11}}$

$\tan\alpha = \frac{\sqrt{7}}{2} \Rightarrow \tan\alpha = \frac{-\sqrt{7}}{2}$

$\tan(\pi-\alpha) = \frac{\sqrt{7}}{2} = -\tan\alpha$

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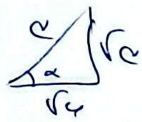
$2\sin^2 x + \cos^2 x = \frac{4}{3} \rightarrow \sin^2 x + 1 = \frac{4}{3}$

$\tan^2 x = ?$

$\sin^2 x = \frac{1}{3} \rightarrow \sin x = \frac{\sqrt{3}}{3}$



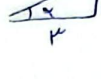
$\tan x = \frac{\sqrt{2}}{2} \rightarrow \tan^2 x = \frac{1}{2}$

$$\frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha} = \frac{\sin^4\alpha + 4 - 4\sin^2\alpha}{2-\sin^2\alpha} - \frac{\cos^4\alpha + 4 - 4\cos^2\alpha}{2-\cos^2\alpha}$$

$1+\cos^2\alpha = 1-\sin^2\alpha$, $1+\sin^2\alpha = 1-\cos^2\alpha$

$$= \frac{(2-\sin^2\alpha)^2}{2-\sin^2\alpha} - \frac{(2-\cos^2\alpha)^2}{2-\cos^2\alpha} = \cancel{4} - \sin^2\alpha - \cancel{4} + \cos^2\alpha = \cos^2\alpha - \sin^2\alpha = \cos 2\alpha$$

$\tan\alpha = \frac{4}{3}$, $\alpha \to$ (3)



$\cos\alpha = \frac{3}{5}$, $\sin\alpha = \frac{4}{5}$

$$\sin\left(\frac{3}{2}\pi + \alpha\right)\cos\left(\frac{7}{2}\pi - \alpha\right) - \tan\left(\frac{3}{2}\pi + \alpha\right) = (\cos\alpha)(-\sin\alpha) - (-\cot\alpha) =$$

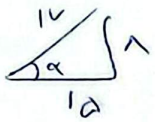
$$= -(\cos\alpha)(\sin\alpha) + (\cot\alpha) = -\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) + \frac{3}{4} = -\frac{12}{25} + \frac{3}{4} = \frac{48-75}{100} = \frac{-27}{100}$$

$$\left(4\cos^2\alpha + \sqrt{2}\sin\alpha - \sqrt{2}\cos\alpha\right)^2 ? \qquad \underbrace{3\cos\left(\frac{\pi}{2}+\alpha\right)}_{-\sin\alpha} + \sqrt{2}\underbrace{\left(\sin\alpha - \cos\alpha\right)}_{\sqrt{2}\sin\left(\alpha-\frac{\pi}{4}\right)}$$

$\alpha = \frac{\pi}{12}$

$$= \left(4\cos^2\frac{\pi}{12} + 2\sin\left(\alpha - \frac{\pi}{4}\right)\right)^2 = \left(4\cos^2\frac{\pi}{12} - 2\sin\frac{\pi}{6}\right)^2 = \left(4\cos^2\frac{\pi}{12} - 1\right)^2 = \left(2\cos\frac{\pi}{6} - 1\right)^2 = \left(\sqrt{3}-1\right)^2 = 4 - 2\sqrt{3}$$

$\tan\left(\frac{\alpha}{2}\right) = \frac{1}{3}$ → $\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} = \frac{\frac{2}{3}}{1-\frac{1}{9}} = \frac{\frac{2}{3}}{\frac{8}{9}} = \frac{3}{4}$ ، $\cos\alpha = \frac{4}{5}$



چون $\tan\alpha$ و $\tan\frac{\alpha}{2}$ هم‌علامت‌اند پس α در ناحیه (1) می‌باشد $\sin\alpha = \frac{3}{5}$

$$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{3}{4} - \frac{3}{5}}{\frac{3}{5} - \frac{4}{5}} = \frac{\frac{15-12}{20}}{\frac{3-4}{5}} = \frac{\frac{3}{20}}{-\frac{1}{5}} = -\frac{3}{4}$$

$\frac{\cot\alpha}{\sin\alpha} > 0 \rightarrow \frac{\cos\alpha}{\sin^2\alpha} > 0 \rightarrow \boxed{\cos\alpha > 0}$

$2\sin\alpha < \sin 2\alpha \rightarrow 2\sin\alpha < 2\sin\alpha\cos\alpha$

$2\sin\alpha - 2\sin\alpha\cos\alpha < 0$

$2\sin\alpha(1-\cos\alpha) < 0$

$0 \quad \downarrow$

$-1 < \cos\alpha < 1$

$0 > 1 - \cos\alpha > 0$

$2 > 1 - \cos\alpha > 0$

این ⊕ است

$\cup_2$

$2\sin\alpha < 0$

$\boxed{\sin\alpha < 0}$

$\downarrow$

④ ناحیه α