هلیا اسفندیاران

مثلث ABC با اضلاع ۶ و $\sqrt{۳}$ و زاویه بین آنها (α)

$S = ۴,۵ \rightarrow S = \frac{1}{2} ab \sin\alpha$

$\frac{\max\alpha}{\min\alpha} = ?$

$۴,۵ = \frac{۶ \times \sqrt{۳}}{۲} \times \sin\alpha \rightarrow \frac{۳}{۲} = \sqrt{۳} \sin\alpha \rightarrow \sin\alpha = \frac{\frac{۳}{۲}}{\sqrt{۳}}$

$\sin\alpha = \frac{\sqrt{۳}}{۲} = \alpha =$  $\rightarrow \sin\alpha$

$\max \alpha = ۱۲۰°$

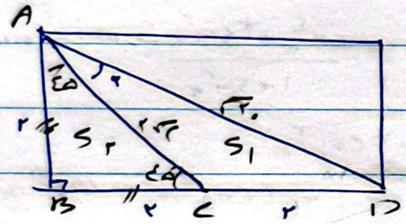
$\min \alpha = ۶۰°$

$\frac{۱۲۰}{۶۰} = ۲$

۲- $\cot\alpha = ?$

مثلث متساوی‌الساقین قائم‌الزاویه ABC ← مثلث ABC

AC میانه است پس ۲ مثلث هم‌مساحت

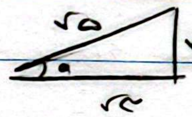


$S_1 = S_2$

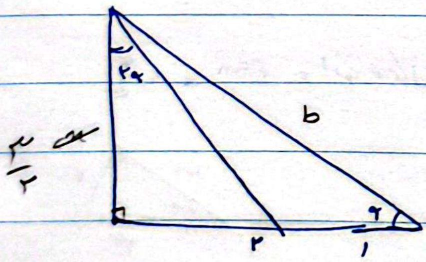
$\frac{1}{2} \times AB \times AC \times \sin 45 = \frac{1}{2} \times AC \times AD \sin\alpha$

$\sqrt{۲} = \sqrt{۵} \sin\alpha$

$\sin\alpha = \frac{\sqrt{۲}}{\sqrt{۵}}$



$\rightarrow \cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{\sqrt{۳}}{\sqrt{۲}} = \sqrt{\frac{۳}{۲}}$



$\cot\alpha = ?$ $\cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{\frac{۳}{b}}{\frac{۳}{۲b}} = ۲$

$\tan 2\alpha = \frac{۲}{x} \rightarrow \tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$

$\tan\alpha = \frac{x}{۳}$

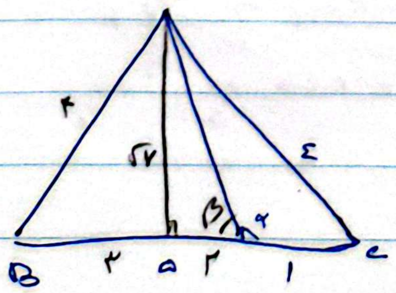
$\frac{۲}{x} = \frac{\frac{۲}{۳}x}{\frac{۹ - x^2}{۹}} \rightarrow ۱۸ - ۲x^2 = ۶x^2 \rightarrow ۸x^2 = ۱۸ \rightarrow x = \sqrt{\frac{۹}{۴}} = \frac{۳}{۲}$

ABC = متساوی‌الساقین

$\tan\alpha = ?$ $\tan\alpha = -\tan\beta$

$\beta = ۱۸۰ - \alpha$

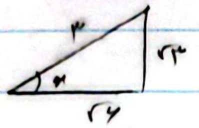
$\tan\beta = \frac{\sin\beta}{\cos\beta} = \frac{\sqrt{۷}}{۳} \rightarrow -\tan\beta = -\frac{\sqrt{۷}}{۳}$



حل المسائل

۵- $\tan^2 x = ? \quad \left(\frac{\sin x}{\cos x}\right)^2 = \left(\frac{\sqrt{2}}{\sqrt{6}}\right)^2 = \left(\frac{\sqrt{3}}{3}\right)^2 = \frac{1}{3}$ $\qquad 2\sin^2 x + \cos^2 x = \frac{4}{3}$

$\sin^2 x + \underbrace{\sin^2 x + \cos^2 x}_{1} = \frac{4}{3} \to \sin^2 x = \frac{1}{3} \to \sin x = \pm\frac{\sqrt{3}}{3}$



۶- ساده شده عبارت $\frac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha} - \frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha}$

$\sin^4 x = (1-\cos^2 x)^2 = 1 - 2\cos^2 x + \cos^4 x \xrightarrow{+4\cos^2 x} 1 + 2\cos^2 x + \cos^4 x = (\cos^2 x + 1)^2$

$\cos^4 x = (1-\sin^2 x)^2 = 1 - 2\sin^2 x + \sin^4 x \xrightarrow{+4\sin^2 x} \sin^4 x + 2\sin^2 x + 1 = (\sin^2 x + 1)^2$

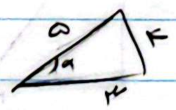
$\frac{(\cos^2 x + 1)^2}{1+\cos^2 x} - \frac{(\sin^2 x + 1)^2}{1+\sin^2 x} = \cos^2 x + 1 - \sin^2 x - 1 = \cos^2 x - \sin^2 x$

$= \cos 2x$

۷- $\tan\alpha = \frac{4}{3}$ و انتهای کمان α در ربع سوم باشد

$\sin\left(\frac{9\pi}{2}+\alpha\right) \cos\left(\frac{7\pi}{2}-\alpha\right) - \tan\left(\alpha - \frac{3\pi}{2}\right)$

$\sin\left(\frac{\pi}{2}+\alpha\right) = \cos\alpha \qquad \frac{\cos\left(\frac{\pi}{2}-\alpha\right)}{\sin\alpha} \qquad +\cot\alpha$



$\cos\alpha = -\frac{3}{5}$

$(\cos\alpha)(\sin\alpha) + \cot\alpha$

$\sin\alpha = -\frac{4}{5}$

$-\frac{3}{5} \times -\frac{4}{5} \times \frac{3}{4} = \frac{9}{25}$

۸- حاصل ۱۴۹ $\quad (3\cos 4x + \sqrt{2}\sin x - \sqrt{2}\cos x) \quad x = \frac{\pi}{12}$

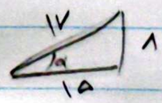
$3\cos 4x + \sqrt{2}(\sin x - \cos x) = 3\cos\left(4 \times \frac{\pi}{12}\right) + 2\sin\left(\frac{\pi}{12} - \frac{\pi}{4}\right) = 2\sin\left(-\frac{\pi}{6}\right)$

$(\sqrt{2}\sin(x - \frac{\pi}{4}))$

$3\cos\frac{\pi}{3} + 2\sin\frac{\pi}{6} = 3 \times \frac{1}{2} + 2 \times \frac{1}{2} = \frac{3}{2} + 1 = \frac{5}{2}$

$\tan\alpha > 0 \to \sin\alpha\cos\alpha > 0 \qquad \tan\left(\frac{\alpha}{2}\right) = \frac{1}{4} \quad ?$

$\frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \frac{\frac{16}{15 \times 17}}{-\frac{7}{17}} = -\frac{2}{15}$ $\qquad \frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha}$



$\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} \to \tan\alpha = \frac{2 \times \frac{1}{4}}{1-\frac{1}{16}} = \frac{\frac{1}{2}}{\frac{15}{16}} = \frac{8}{15}$

حل المسائل

۱۵- $\sin 2\alpha < 2\sin\alpha$ و $\frac{\cot\alpha}{\sin\alpha} > 0$ ، انتهای کمان α در کدام ربع است؟ ربع چهارم

$2\sin\alpha > 2\sin\alpha\cos\alpha$

$-1 \le \cos\alpha \le 1$

$0 < 2\sin\alpha(\cos\alpha - 1)$ — $\sin\alpha$: $(-)$ ، $\cos\alpha - 1$: $(-)$

$\frac{\cos\alpha}{\sin\alpha} > 0$ ← مثبت بوده است

$\sin\alpha$ همراه $(+)$