گروه یازدهم (دختران)　　۱۹　　زهرا آقایی

۱

$S = \frac{1}{2} \times 6 \times \sqrt{3} \times \sin\alpha = 4.5$　　۲

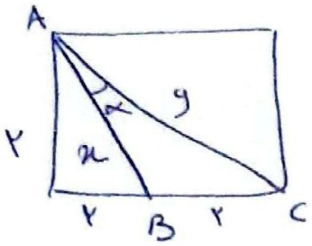
$\sin\alpha = \frac{\sqrt{3}}{2} \to \alpha = 60 , 120$

$\frac{120}{60} = 2$

۲) $\tan(45+\alpha) = 2 = \frac{\tan\alpha + 1}{1-\tan\alpha} \to \tan\alpha = \frac{1}{3}$

$\to \cot\alpha = 3$

۳



$x = 2\sqrt{2}$, $y = 2\sqrt{5}$ } → از روش فیثاغورس

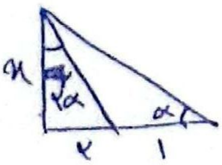
$\overset{\triangle}{ABC} = BC = \sqrt{x^2 + y^2 - 2xy\cos\alpha}$　　۱

$4 = 8 + 20 - \sqrt{10}\cos\alpha$

$\boxed{\cos\alpha = \frac{3}{\sqrt{10}}}$

$\to \sin\alpha = \frac{1}{\sqrt{10}}$ ، $\cot\alpha = 3$

۴



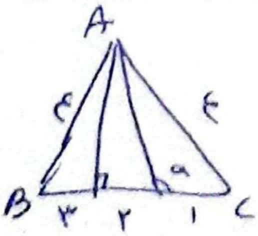
$\cot\alpha = \frac{3}{x} \to \tan x = \frac{x}{3}$　　$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$

$\tan 2\alpha = \frac{2}{x}$

$\frac{2}{x} = \frac{\frac{2x}{3}}{1-\frac{x^2}{9}} \to 2 - \frac{2x^2}{9} = \frac{2x^2}{3}$　　۶

$\frac{4x^2}{9} = 1 \to x = \pm\frac{3}{2}$　　$\cot\alpha = \frac{3}{\frac{3}{2}} = \boxed{2}$

۵



$AH = \sqrt{16-9} = \sqrt{7}$

$\tan(\pi - \alpha) = \frac{\sqrt{7}}{3} \to \tan\alpha = \frac{\sqrt{7}}{3}$

$\tan\alpha = -\frac{\sqrt{7}}{3}$　　۶

۶

$2\sin^2 x + \cos^2 x = \sin^2 x + \underbrace{\sin^2 x + \cos^2 x}_{1} = \frac{4}{3}$　　$\sin^2 x = \frac{1}{3}$

$1 + \tan^2 = \frac{1}{\cos^2} \to \frac{3}{2} - 1 = \boxed{\frac{1}{2}}$　　۶　$\cot^2 x = \frac{2}{3}$

6

$$\frac{\sin^4\alpha+4\cos^2\alpha}{1+\cos^2\alpha}-\frac{\cos^4\alpha+4\sin^2\alpha}{1+\sin^2\alpha}$$

$$\sin^4\alpha=(\sin^2\alpha)^2=(1-\cos^2\alpha)^2\Rightarrow 1+\cos^4\alpha-2\cos^2\alpha$$

$$\frac{1+\cos^4\alpha\underbrace{-2\cos^2\alpha+4\cos^2\alpha}_{+2\cos^2\alpha}}{1+\cos^2\alpha}-\frac{1+\sin^4\alpha\underbrace{\cancel{-2\sin^2\alpha+4\sin^2\alpha}}_{2\sin^2\alpha}}{1+\sin^2\alpha}=$$

$$\frac{(\cos^2\alpha+1)^2}{1+\cos^2\alpha}=-\frac{(\sin^2\alpha+1)^2}{1+\sin^2\alpha}\to\cos^2\alpha+1-\sin^2\alpha-1=\cos^2\alpha-\sin^2\alpha$$

$$\boxed{\cos2\alpha}$$

7

$$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right)-\tan\left(\alpha-\frac{3\pi}{2}\right)=\cos\alpha\times(-\sin\alpha)+$$

$$\cot\alpha=\overbrace{\left(\frac{-3}{5}\times\frac{4}{5}\right)}^{-\frac{12}{25}}+\frac{3}{4}=\frac{27}{100}$$

$\sin\alpha=-\frac{4}{5}$ $\cot\alpha=$

$\cos\alpha=-\frac{3}{5}$

8

$$\sin^2\frac{\pi}{12}=\frac{1-\cos\frac{\pi}{6}}{2}=\frac{1-\frac{\sqrt3}{2}}{2}\Rightarrow\sin\frac{\pi}{12}=\frac{\sqrt{2-\sqrt3}}{2}$$

$$\cos^2\frac{\pi}{12}=\frac{1+\cos\frac{\pi}{6}}{2}=\frac{1+\frac{\sqrt3}{2}}{2}\to\cos\frac{\pi}{12}=\frac{\sqrt{2+\sqrt3}}{2}$$

$$3\cos\frac{\pi}{3}+\sqrt2\left(\frac{\sqrt{2-\sqrt3}}{2}\right)-\sqrt2\left(\frac{\sqrt{2+\sqrt3}}{2}\right)=\frac{3}{2}+\frac{\sqrt{(1-\sqrt3)^2}}{2}-\frac{\sqrt{(1+\sqrt3)^2}}{2}$$

$$\frac{3+\sqrt3-1-\sqrt3-1}{2}=\frac{1}{2}$$

9

$$\tan^2\left(\frac{\alpha}{2}\right)=\frac{1-\cos\alpha}{1+\cos\alpha}\Rightarrow\frac{1}{16}=\frac{1-\cos\alpha}{1+\cos\alpha}$$

$$17\cos\alpha=15\to\cos\alpha=\frac{15}{17}$$

$\sin\alpha=\frac{8}{17}$

$\tan\alpha=\frac{8}{15}$

$$\frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha}=\frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}}=-\frac{16}{105}$$

۱۰

$\cancel{2\sin\alpha} < 2\cancel{\sin\alpha}\cos\alpha$ → $\cos\alpha > 1$ → غ ق ق

$2\sin\alpha < \sin 2\alpha$ → $\cos\alpha < 1$ ✓ → $2\sin\alpha < 0$

$\sin\alpha < 0$ ، $\dfrac{\cot\alpha}{\sin\alpha} > 0$ → $\cot\alpha < 0$ → α در ناحیه چهارم