گروه یازدهم (دختران)

زهرا آقایی

(۱)

$S=\frac{1}{2}\times 6\times\sqrt{3}\times\sin\alpha=4.5$

$\sin\alpha=\frac{\sqrt{3}}{2}\rightarrow\alpha=60\,,\,120$

$\frac{120}{60}=2$

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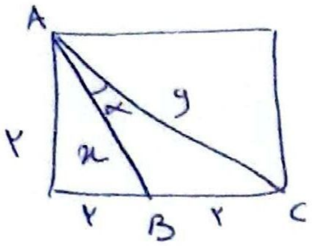

$\left.\begin{array}{l}x=2\sqrt{2}\\ y=2\sqrt{5}\end{array}\right\}\rightarrow$ از روش فیثاغورس

$\overset{\Delta}{ABC}=BC=\sqrt{x^2+y^2-2xy\cos\alpha}$

$\leftarrow 4=8+20-\sqrt{10}\cos\alpha$

$\boxed{\cos\alpha=\frac{3}{\sqrt{10}}}$

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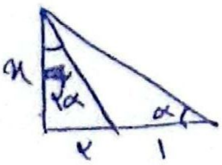


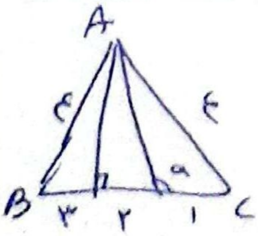
$\cot\alpha=\frac{3}{x}\rightarrow\tan\alpha=\frac{x}{3}\qquad\tan2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}$

$\tan2\alpha=\frac{2}{x}$

$\frac{2}{x}=\frac{\frac{2x}{3}}{1-\frac{x^2}{9}}\rightarrow 2-\frac{2x^2}{9}=\frac{2x^2}{3}$

$\frac{4x^2}{9}=1\rightarrow x=\pm\frac{3}{2}\qquad\cot\alpha=\frac{3}{\frac{3}{2}}=2$

(۴)



$AH=\sqrt{16-9}=\sqrt{7}$

$\tan(\pi-\alpha)=\frac{\sqrt{7}}{2}\rightarrow\tan\alpha=\frac{\sqrt{7}}{2}$

$\tan\alpha=-\frac{\sqrt{7}}{2}$

(۵)

$2\sin^2x+\cos^2x=\sin^2x+\underbrace{\sin^2x+\cos^2x}_{1}=\frac{4}{3}$

$\sin^2x=\frac{1}{3}$

$\cos^2x=\frac{2}{3}$

$1+\tan^2=\frac{1}{\cos^2}\rightarrow\frac{3}{2}-1=\frac{1}{2}$

(6)

$$\frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha}$$

$\sin^4\alpha = (\sin^2\alpha)^2 = (1-\cos^2\alpha)^2 \Rightarrow 1+\cos^4\alpha - 2\cos^2\alpha$

$$\frac{1+\cos^4\alpha - 2\underbrace{\cos^2\alpha + 4\cos^2\alpha}_{+2\cos^2\alpha}}{1+\cos^2\alpha} - \frac{1+\sin^4\alpha \underbrace{- \cancel{2\sin^2\alpha} + 4\sin^2\alpha}_{2\sin^2\alpha}}{1+\sin^2\alpha} =$$

$$\frac{(\cos^2\alpha+1)^2}{1+\cos^2\alpha} = -\frac{(\sin^2\alpha+1)^2}{1+\sin^2\alpha} \to \cos^2\alpha + 1 - \sin^2\alpha - 1 = \cos^2\alpha - \sin^2\alpha$$

$\boxed{\cos 2\alpha}$

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$$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right) - \tan\left(\alpha-\frac{3\pi}{2}\right) = \cos\alpha \times (-\sin\alpha) +$$

$$\cot\alpha = \underbrace{\left(\frac{-3}{5}\times\frac{4}{5}\right)}_{-\frac{12}{25}} + \frac{3}{4} = \frac{27}{100}$$

$\sin\alpha = \frac{4}{5}$ $\cot\alpha$

$\cos\alpha = \frac{-3}{5}$

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$$\sin^2\frac{\pi}{12} = \frac{1-\cos\frac{\pi}{6}}{2} = \frac{1-\frac{\sqrt3}{2}}{2} \Rightarrow \sin\frac{\pi}{12} = \frac{\sqrt{2-\sqrt3}}{2}$$

$$\cos^2\frac{\pi}{12} = \frac{1+\cos\frac{\pi}{6}}{2} = \frac{1+\frac{\sqrt3}{2}}{2} \to \cos\frac{\pi}{12} = \frac{\sqrt{2+\sqrt3}}{2}$$

$$3\cos\frac{\pi}{3} + \sqrt2\left(\frac{\sqrt{2-\sqrt3}}{2}\right) - \sqrt2\left(\frac{\sqrt{2+\sqrt3}}{2}\right) = \frac{3}{2} + \frac{\sqrt{(1-\sqrt3)^2}}{2} - \frac{\sqrt{(1+\sqrt3)^2}}{2}$$

$$\frac{3+\sqrt3-1-\sqrt3-1}{2} = \frac{1}{2}$$

(9)

$$\tan^2\left(\frac{\alpha}{2}\right) = \frac{1-\cos\alpha}{1+\cos\alpha} \Rightarrow \frac{1}{16} = \frac{1-\cos\alpha}{1+\cos\alpha}$$

$17\cos\alpha = 15 \to \cos\alpha = \frac{15}{17}$

$\sin\alpha = \frac{8}{17}$

$\tan\alpha = \frac{8}{15}$

$$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}} = \frac{-16}{105}$$

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$$\left.\begin{array}{l} 2\cancel{\sin\alpha} < 2\cancel{\sin\alpha}\cos\alpha \\ 2\sin\alpha < \sin 2\alpha \end{array}\right\} \begin{array}{l} \to \cos\alpha > 1 \to \text{غ ق ق} \\ \to \cos\alpha < 1 \checkmark \longrightarrow 2\sin\alpha < 0 \end{array}$$

$$\sin\alpha < 0 \quad , \quad \frac{\cot\alpha}{\sin\alpha} > 0 \to \cot\alpha < 0 \longrightarrow \alpha \text{ در ناحیه چهارم}$$