نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۲۷ کلاس .....

(۲۰)

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$\cos\alpha = \frac{1}{2} \times \sin\alpha \times \frac{2}{\sqrt{2}\alpha} = \frac{\sqrt{2}}{2} \rightarrow \sin\alpha = \frac{\sqrt{2}}{2}$

$\alpha = 45^\circ$ (min) ، $\alpha = 135^\circ$ (max)

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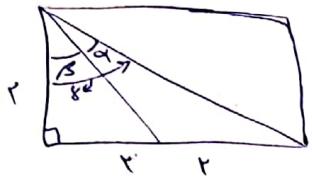

$\tan\alpha = \tan(\gamma - \beta) = \frac{\tan\gamma - \tan\beta}{1 + \tan\gamma\tan\beta} = \frac{2 - \frac{1}{2}}{1 + 2 \times \frac{1}{2}} = \frac{\frac{3}{2}}{3} = \frac{1}{2}$

$\rightarrow \cot\alpha = 2$

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$\tan 2\alpha = \frac{2\tan\alpha}{1 - (\tan\alpha)^2} \Rightarrow \frac{x}{2} = \frac{2 \times \frac{1}{2}}{1 - \frac{1}{4}} \Rightarrow \frac{1}{x} = \frac{\frac{1}{2}}{\frac{3}{4}} \Rightarrow \frac{1}{x} = \frac{2}{3} \rightarrow 2x^2 = 3 - x^2$

$3x^2 = 3 \Rightarrow x = 1$

$\cot\alpha = \frac{2}{1} = 2$

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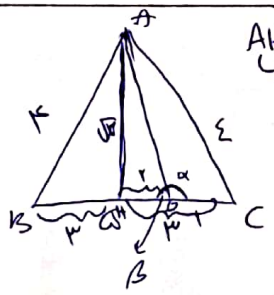

$AH^2 = 14 - 9 = 5 \rightarrow AH = \sqrt{5}$

$\tan\beta = \frac{AH}{HB} = \frac{\sqrt{5}}{2} \rightarrow \beta$ از جدول

$= -\tan\beta = -\frac{\sqrt{5}}{2}$

$\tan(\alpha) = \tan(2\alpha - \beta)$

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$2\sin^2 x + \cos^2 x = \frac{4}{3}$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = \frac{1}{3} \rightarrow \cos^2 x = \frac{2}{3}$

$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$

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$$\frac{\sin^4\alpha+4\cos^2\alpha}{1+\cos^2\alpha}-\frac{\cos^4\alpha+4\sin^2\alpha}{1+\sin^2\alpha}=\frac{\sin^4\alpha+4-4\sin^2\alpha}{1+\cos^2\alpha}-\frac{\cos^4\alpha+4-4\cos^2\alpha}{1+\sin^2\alpha}$$

$$=\frac{(2-\sin^2\alpha)^2}{1+\cos^2\alpha}-\frac{(2-\cos^2\alpha)^2}{1+\sin^2\alpha}=\frac{(2-\sin^2\alpha)^2}{2-\sin^2\alpha}-\frac{(2-\cos^2\alpha)^2}{2-\cos^2\alpha}$$

$$=2-\sin^2\alpha-2+\cos^2\alpha=\cos^2\alpha-\sin^2\alpha=\cos 2\alpha$$

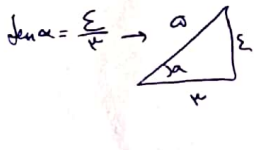
۷

$$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right)-\tan(\alpha-\frac{3\pi}{2})=\cos\alpha\times(-\sin\alpha)+\cot\alpha=\frac{-3}{5}\times\left(\frac{4}{5}\right)+\frac{3}{4}$$

$$\sin\left(4\pi+\frac{\pi}{2}+\alpha\right)=\sin\left(\frac{\pi}{2}+\alpha\right)=\cos\left(\frac{3\pi}{2}-\frac{\pi}{2}-\alpha\right)=\cos\left(-\frac{\pi}{2}-\alpha\right)=\cos\left(\alpha+\frac{\pi}{2}\right)$$

$$\tan(\alpha-\frac{3\pi}{2})=-\tan(\frac{3\pi}{2}-\alpha)=-\cot\alpha$$

$$=\frac{-12}{20}+\frac{3}{4}=\frac{-24+15}{100}=\frac{27}{100}$$



$\sin\alpha=\frac{-4}{5}$

$\cos\alpha=\frac{-3}{5}$

$\tan\alpha=\frac{3}{4}$

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$$3\cos 4x+\sqrt{2}\sin x-\sqrt{2}\cos x=3\cos\frac{4\pi}{12}+(-1)=3\times\frac{1}{2}-1=\frac{1}{2}$$

$$\sqrt{2}(\sin x-\cos x)=\sqrt{2}\times\sqrt{2}\left(\sin\left(x-\frac{\pi}{4}\right)\right)=2\sin\frac{\pi}{4}\geq 2\times\frac{1}{2}\leq 1$$

$$\frac{\pi}{12}-\frac{\pi}{4}=-\frac{\pi}{6}$$

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$$\tan\left(\frac{\alpha}{2}\right)=\frac{1}{4}$$

$$\tan^2\left(\frac{\alpha}{2}\right)=\frac{1-\cos\alpha}{1+\cos\alpha}>\frac{1}{16}>\frac{1-\cos\alpha}{1+\cos\alpha}\Rightarrow 16\cos\alpha=16-16\cos\alpha$$

$$17\cos\alpha=15\rightarrow\cos\alpha=\frac{15}{17}$$

$$\frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha}=\frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}}=\frac{\frac{16}{15\times 17}}{\frac{-119}{17\times 17}}=\frac{-16}{105}$$

$\sin\alpha=\frac{8}{17}$

$\tan\alpha=\frac{8}{15}$

۱۰

$$2\sin\alpha<\sin 2\alpha\rightarrow 2\sin\alpha<2\sin\alpha\cos\alpha\rightarrow 2\sin\alpha(\cos\alpha-1)>0$$

(مثبت) (منفی) (منفی)

$$0<\frac{\cot\alpha}{\sin\alpha}\rightsquigarrow\frac{\frac{\cos\alpha}{\sin\alpha}}{\sin\alpha}>0\rightarrow\frac{\cos\alpha}{\sin^2\alpha}>0$$

(مثبت) (مثبت)

$\cos\alpha$ مثبت
$\sin\alpha$ منفی
$\}\rightarrow$ ربع سوم