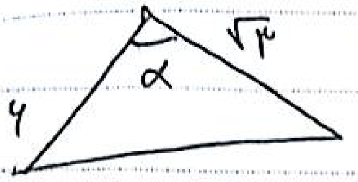
Year: Mounth: Date: (۵) / پنجشنبه یازدهم دفتر

تکلیف ۲۷

Subject : ([illegible])

۲۰

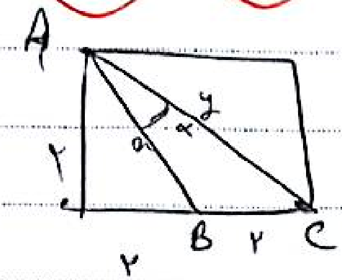
①



$S=\frac{۱}{۲}\times ۲\times\sqrt{۳}\times\sin\alpha=۱٫۵$

$\sin\alpha=\frac{\sqrt{۳}}{۲}\longrightarrow\alpha=۶۰ \text{ یا } ۱۲۰\longrightarrow\frac{۱۲۰}{۶۰}=\boxed{۲}$

②



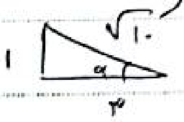
$x=۲\sqrt{۲}$ }
$y=۲\sqrt{۵}$ } فیثاغورث

$A\hat{B}C$: $BC=\sqrt{x^۲+y^۲-۲xy\cos\alpha}$

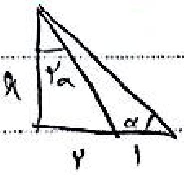
$۴=۸+۲۰-\sqrt{۱۰}\cos\alpha$

$\cos\alpha=\frac{۳}{\sqrt{۱۰}}$

$\boxed{\cot\alpha=۳}$



③



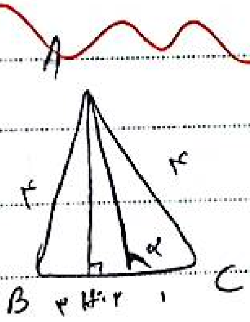
$\cot\alpha=\frac{۳}{x}\longrightarrow\tan x=\frac{x}{۳}$ $\qquad \tan ۲\alpha=\frac{۲\tan\alpha}{۱-\tan^۲\alpha}$

$\tan ۲\alpha=\frac{۲}{x}$

$\frac{۲}{x}=\frac{\frac{۲x}{۳}}{۱-\frac{x^۲}{۹}}\longrightarrow ۲-\frac{۲x^۲}{۹}=\frac{۲x^۲}{۳}$ $\qquad \frac{۴x^۲}{۹}=۱\to x=\pm\frac{۳}{۲}$

$\cot\alpha=\frac{۳}{\frac{۳}{۲}}=\boxed{۲}$

④



$AH\ \sqrt{۱۶-۹}=\sqrt{۷}$

$\tan(\pi-\alpha)=\frac{\sqrt{۷}}{۲}\to\tan\alpha=\frac{\sqrt{۷}}{۲}$

$\boxed{\tan\alpha=-\frac{\sqrt{۷}}{۲}}$

⑤

$۲\sin^۲x+\cos^۲x=\sin^۲x+\underbrace{\sin^۲x+\cos^۲x}_{۱}=\frac{۴}{۳}\to\sin^۲x=\frac{۱}{۳}$

$\cos^۲x=\frac{۲}{۳}$

$۱+\tan^۲x=\frac{۱}{\cos^۲x}\to\tan^۲x=\frac{۳}{۲}-۱=\boxed{\frac{۱}{۲}}$

⑥

$$\frac{\sin^۴\alpha+۴\cos^۲\alpha}{۱+\cos^۲\alpha}-\frac{\cos^۴\alpha+۴\sin^۲\alpha}{۱+\sin^۲\alpha}$$

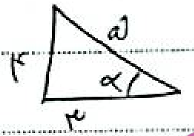
$$\sin^۴\alpha=(\sin^۲\alpha)^۲=(۱-\cos^۲\alpha)^۲=۱+\cos^۴\alpha-۲\cos^۲\alpha$$

$$\frac{۱+\cos^۴\alpha\underset{۲\cos^۲\alpha}{\cancel{-۲\cos^۲\alpha+۴\cos^۲\alpha}}}{۱+\cos^۲\alpha}-\frac{۱+\sin^۴\alpha\underset{۲\sin^۲\alpha}{\cancel{-۲\sin^۲\alpha+۴\sin^۲\alpha}}}{۱+\sin^۲\alpha}=\frac{(\cos^۲\alpha+۱)^۲}{۱+\cos^۲\alpha}-\frac{(\sin^۲\alpha+۱)^۲}{۱+\sin^۲\alpha}=$$

$$\cos^۲\alpha+۱-\sin^۲\alpha-۱=\cos^۲\alpha-\sin^۲\alpha=\boxed{\cos ۲\alpha}$$

⑦

$$\sin\left(\frac{۹\pi}{۲}+\alpha\right)\cos\left(\frac{۷\pi}{۲}-\alpha\right)-\tan\left(\alpha-\frac{۳\pi}{۲}\right)=$$



$\sin\alpha=-\frac{۴}{۵}$

$\cos\alpha=-\frac{۳}{۵}$

$\cot=\frac{۳}{۴}$

$$\cos\alpha\times(-\sin\alpha)+\cot\alpha=\overset{-\frac{۱۲}{۲۵}}{\left(\frac{-۳}{۵}\times\frac{۴}{۵}\right)}+\frac{۳}{۴}=\boxed{\frac{۲۷}{۱۰۰}}$$

⑧

$$\sin^۲\frac{\pi}{۱۲}=\frac{۱-\cos\frac{\pi}{۶}}{۲}=\frac{۱-\frac{\sqrt{۳}}{۲}}{۲}\to\sin\frac{\pi}{۱۲}=\frac{\sqrt{۲-\sqrt{۳}}}{۲}$$

$$\cos^۲\frac{\pi}{۱۲}=\frac{۱+\cos\frac{\pi}{۶}}{۲}=\frac{۱+\frac{\sqrt{۳}}{۲}}{۲}\longrightarrow\cos\frac{\pi}{۱۲}=\frac{\sqrt{۲+\sqrt{۳}}}{۲}$$

$$۳\cos\frac{\pi}{۳}+\sqrt{۲}\left(\frac{\sqrt{۲-\sqrt{۳}}}{۲}\right)-\sqrt{۲}\left(\frac{\sqrt{۲+\sqrt{۳}}}{۲}\right)=\frac{۳}{۲}+\frac{\sqrt{(۱-\sqrt{۳})^۲}}{۲}-\frac{\sqrt{(۱+\sqrt{۳})^۲}}{۲}$$

$$=\frac{۳+\sqrt{۳}-۱-\sqrt{۳}-۱}{۲}=\boxed{+\frac{۱}{۲}}$$

۹

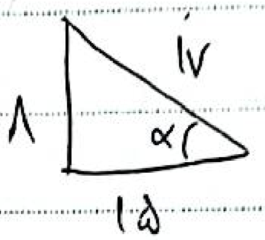
$\tan^۲(\frac{\alpha}{۲}) = \frac{1-\cos\alpha}{1+\cos\alpha} \rightarrow \frac{1}{۱۶} = \frac{1-\cos\alpha}{1+\cos\alpha}$

$۱۷\cos\alpha = ۱۵$

$\cos\alpha = \frac{۱۵}{۱۷}$

$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{۸}{۱۵} - \frac{۸}{۱۷}}{\frac{۸}{۱۷} - \frac{۱۵}{۱۷}} = \frac{\frac{۱۶}{۲۵۵}}{\frac{-۷}{\cancel{۱۷}}}$

$\boxed{= -\frac{۱۶}{۱۰۵}}$



$\sin\alpha = \frac{۸}{۱۷}$

۹

$\tan\alpha = \frac{۸}{۱۵}$

۱۰

$۲\sin\alpha < \sin۲\alpha$

$\cancel{۲\sin\alpha} < ۲\sin\alpha\cancel{\cos\alpha}$

$\cos\alpha > ۱ \rightarrow$ غقق

$\cos\alpha < ۱$ ✓ $\rightarrow$ پس $۲\sin\alpha < ۰$

۹

$\sin\alpha < ۰$ ، $\frac{\cot\alpha}{\sin\alpha} > ۰ \rightarrow \cot\alpha < ۰ \rightarrow$ $\boxed{\text{ربع چهارم}}$