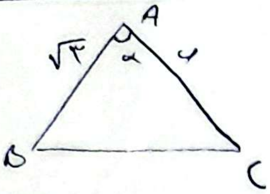
باران کاظمی — تکلیف شماره‌ی ۲۷

۱۹,۲۵

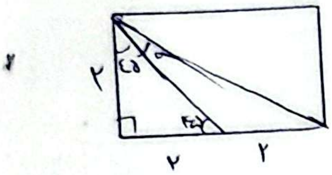
۱)



$$\frac{1}{2}\times\sqrt{3}\times 6\times\sin\alpha = 4,5 \to \sin\alpha = \frac{9}{2\times 6\sqrt{3}} = \frac{3\sqrt{3}}{2\times 3} = \frac{\sqrt{3}}{2}$$

$$\sin\alpha = \frac{\sqrt{3}}{2} \begin{cases} \alpha = 60 \\ \alpha = 120 \end{cases} \to \frac{120}{60} = \boxed{2}$$

۲)



$$\cos(45+\alpha) = \cos45\cos\alpha - \sin45\sin\alpha = \frac{2}{2\sqrt{5}}$$

$$\frac{\sqrt{2}}{2}(\cos\alpha - \sin\alpha) = \frac{2}{2\sqrt{5}} \to \cos\alpha - \sin\alpha = \frac{2\sqrt{5}}{5\sqrt{2}} \quad (1)$$

$$\sin(45+\alpha) = \sin45\cos\alpha + \sin\alpha\cos45 = \frac{\sqrt{2}}{2}(\cos\alpha+\sin\alpha) = \frac{4}{2\sqrt{5}}$$

$$\to \cos\alpha + \sin\alpha = \frac{4\sqrt{5}}{5\sqrt{2}} \quad (2)$$

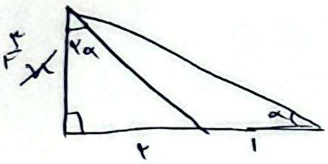
$$(1),(2) \to \begin{cases} \cos\alpha - \sin\alpha = \frac{2\sqrt{5}}{5\sqrt{2}} \\ \cos\alpha + \sin\alpha = \frac{4\sqrt{5}}{5\sqrt{2}} \end{cases} \to \sin\alpha = \frac{\sqrt{5}}{5\sqrt{2}}$$

$$2\cos\alpha = \frac{6\sqrt{5}}{5\sqrt{2}} \to \cos\alpha = \frac{3\sqrt{5}}{5\sqrt{2}}$$

$$\boxed{\cot\alpha = 3}$$

۳)

۱,۷۵



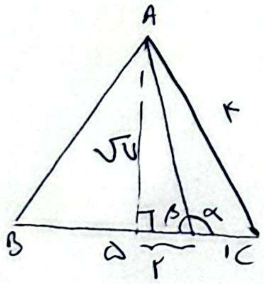
$$\tan2\alpha = \frac{2}{x} \qquad \tan\alpha = \frac{x}{3}$$

$$\tan2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} \to \frac{2}{x} = \frac{\frac{2x}{3}}{1-\frac{x^2}{9}} \Rightarrow \frac{2}{x} = \frac{\frac{2x}{3}}{\frac{9-x^2}{9}}$$

$$\to \frac{2}{x} = \frac{6x}{9-x^2} \to 6x^2 = 18 - 2x^2 \to 8x^2 = 18 \to x^2 = \frac{9}{4} \to x = \frac{3}{2}$$

$$\tan\alpha = \frac{\frac{3}{2}}{3} = \boxed{\frac{1}{2}} \to \cot\alpha = 2$$

۴)



$$\tan\alpha = \tan(180-\beta) = -\tan\beta = \boxed{-\frac{\sqrt{7}}{2}}$$

۵)

۱,۵

$$\begin{cases} 2\sin^2x + \cos^2x = \frac{4}{3} \\ \sin^2x + \cos^2x = 1 \end{cases} \to \cos^2x = \frac{2}{3}$$

$$\sin^2x = \frac{1}{3}$$

$$\tan^2x = \frac{\sin^2x}{\cos^2x} = \frac{\frac{1}{3}}{\frac{2}{3}} = \boxed{2}$$

$$\frac{\frac{1}{3}}{\frac{2}{3}} = 2$$

(۶

$$\frac{\sin^4\alpha+4\cos^2\alpha}{1+\cos^2\alpha}-\frac{\cos^4\alpha+4\sin^2\alpha}{1+\sin^2\alpha}=\frac{\sin^4\alpha+4(1-\sin^2\alpha)}{1+1-\sin^2\alpha}-\frac{\cos^4\alpha+4(1-\cos^2\alpha)}{1+1-\cos^2\alpha}$$

$$=\frac{\sin^4\alpha-4\sin^2\alpha+4}{2-\sin^2\alpha}-\frac{\cos^4\alpha-4\cos^2\alpha+4}{2-\cos^2\alpha}=\frac{(\sin^2\alpha-2)^{\cancel{2}}}{-\cancel{(\sin^2\alpha-2)}}-\frac{(\cos^2\alpha-2)^{\cancel{2}}}{-\cancel{(\cos^2\alpha-2)}}$$

$$=2-\sin^2\alpha-2+\cos^2\alpha=\cos^2\alpha-\sin^2\alpha=\boxed{\cos 2\alpha}$$

(۷

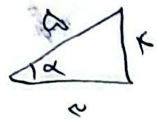
$$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right)-\tan\left(\alpha-\frac{3\pi}{2}\right)$$

$$=\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right)+\tan\left(\frac{3\pi}{2}-\alpha\right)$$

$$=(\cos\alpha)(-\sin\alpha)+(\cot\alpha)$$

$$=\left(-\frac{3}{5}\right)(-)\left(-\frac{4}{5}\right)+\frac{3}{4}$$

$$=-\frac{12}{25}+\frac{3}{4}=\frac{-48+75}{100}=\boxed{0.27}$$



(۸

$$3\cos 4x+\sqrt{2}\sin x-\sqrt{2}\cos x=3\cos\frac{\pi}{3}+\sqrt{2}\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)-\sqrt{2}\cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$$

$$=3\left(\frac{1}{2}\right)+\underbrace{\sqrt{2}\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)}_{\cancel{\frac{\sqrt{12}}{4}}-\frac{1}{2}}-\underbrace{\sqrt{2}\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)}_{-\cancel{\frac{\sqrt{12}}{4}}-\frac{1}{2}}=\frac{3}{2}-\frac{2}{2}=\boxed{\frac{1}{2}}$$

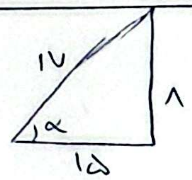
$$\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=\sin\frac{\pi}{3}\cos\frac{\pi}{4}-\sin\frac{\pi}{4}\cos\frac{\pi}{3}=\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)-\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)=\frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=\cos\frac{\pi}{3}\cos\frac{\pi}{4}+\sin\frac{\pi}{3}\sin\frac{\pi}{4}=\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)=\frac{\sqrt{6}+\sqrt{2}}{4}$$

(۹

$$\tan\left(\frac{\alpha}{2}\right)=\frac{1}{4}$$

$$\tan\alpha=\frac{2\tan\left(\frac{\alpha}{2}\right)}{1-\tan^2\left(\frac{\alpha}{2}\right)}=\frac{\frac{1}{2}}{\frac{15}{16}}=\frac{8}{15}$$



$$\frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}}=\frac{\frac{136-120}{255}}{-\frac{7}{17}}=\frac{\frac{16}{15}}{-\frac{7}{1}}=\boxed{-\frac{16}{105}}$$

(۱۰

$$\frac{\cos\alpha}{\underset{+}{\sin 2\alpha}}>0 \qquad \underline{\cos\alpha>0}$$

$$2\sin\alpha<\sin 2\alpha\longrightarrow 0<2\sin\alpha\,(\underbrace{\cos\alpha-1}_{\le 0})$$

$$\Downarrow$$

$$\underline{\sin\alpha<0}$$

ناحیه ۴